

Grid Residual Tropospheric Corrections for Improved Differential GPS Positioning Over the Victoria GPS Network (GPSnet)

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Abstract. Tropospheric delay is one of the major error sources in GPS positioning. The delay of radio signals caused by the troposphere can range from 2 m at the zenith to 20 m at lower elevation angles. In a wide area differential system, tropospheric delays are corrected locally by users using an empirical tropospheric model, with or without meteorological observations. This can easily result in residual tropospheric errors of several centimetres to a few decimetres in positioning solutions. In this paper, the residuals between GPS-derived zenith tropospheric delays (ZTD) and model-computed ZTDs at reference stations of a continuously operating network are obtained. From these residuals, grid residual ZTDs are generated over the network coverage through Ordinary Kriging interpolation. Users can obtain the additional residual corrections for troposphere errors for improved differential positioning over a regional area. Experimental data from one week ZTD estimates from 17 GPSnet reference stations were analysed. Results show that the RMS ZTD accuracy of about 1cm is generally achievable over the Land Victoria GPS network coverage, using the proposed grid tropospheric correction strategies, which can support centimetre level positioning in the region.

Key words: Residual zenith tropospheric delay, Ordinary Kriging, Tropospheric grid model, Wide Area DGPS

1 Introduction

Tropospheric delay is currently one of the major error sources in satellite-based positioning. The delay of radio signals caused by the troposphere can range from 2 m at the zenith to 20 m at lower elevation angles (e.g. below 10 degrees), depending on the pressure, temperature and humidity along the path of the signal transmission. If the

tropospheric delay is not properly modeled, positioning accuracy can be degraded significantly.

In the current code-based Wide Area Differential GPS (WADGPS) positioning services, unlike ionospheric corrections, the tropospheric delay corrections are not broadcast to the users. Instead, the delays are supposed to be corrected locally by using an empirical tropospheric model and a mapping function adopted by the users. Wide Area Augmentation System (WAAS) and European Geostationary Navigation Overlay System (EGNOS) specifications recommend the application of an empirical model correction algorithm, based on the estimates of five meteorological parameters: pressure, temperature, water vapour pressure, temperature lapse rate and water vapour lapse rate. These meteorological parameter estimates are dependent on the receiver's height, latitude and day-of-year, using yearly average and associated seasonal variation data (Collins et al., 1997, Dodson et al., 1999). However, the residual delays after modelling are at a level of a few centimetres at the zenith, which may lead to a single point positioning (SPP) error of up to a few decimetres.

GPS geodetic processing techniques have enabled precise estimation of zenith tropospheric delays (ZTD) with an accuracy on the order of 1 mm for every 30 minutes. Recent studies have demonstrated that the relative ZTDs over a regional network can be estimated in real time (Fang et al., 2003). This provides the technical possibility of using real time ZTD solutions for real time, or near real time precise positioning.

The precise ZTDs estimates derived from ground-based GPS measurements are considered as the "true delays", while the differences between the values computed from a tropospheric delay model and GPS-derived ZTD estimates are defined as the "residual ZTD" estimates. The remaining ZTD model errors are usually ignored by the current WADGPS services. We propose to interpolate the residual ZTDs estimated from reference stations of a

GPS network to generate a residual tropospheric correction grid. The users within the coverage of the network can interpolate residual ZTD from the grid and add it to the ZTD calculated by the empirical model locally.

In our previous research, the ZTD estimates from a network of 129 International GPS Service (IGS) sites across Europe for over 3 months were collected, and the residual ZTDs of each station were directly interpolated by Ordinary Kriging (OK) method using the residual ZTDs from all other stations. From the statistical analysis of each station, the results have concluded that interpolating residual zenith tropospheric delays could be an efficient way to improve user-end ZTD estimation and the precision of differential GPS positioning (Zheng, 2004).

In this paper, additional efforts are made to apply the proposed strategy to the Victoria GPS Network (GPSnet). Further to the previous work, this research will use OK method to interpolate the residual ZTDs of GPSnet stations and to generate a residual tropospheric correction grid at the first place, then obtain the residual ZTDs at given stations interpolated from the grid using the user interpolation algorithm. The efficiency and accuracy of the interpolation procedures will also be analysed.

2 GPS ZTD Estimation

2.1 Victoria GPS Network (GPSnet)

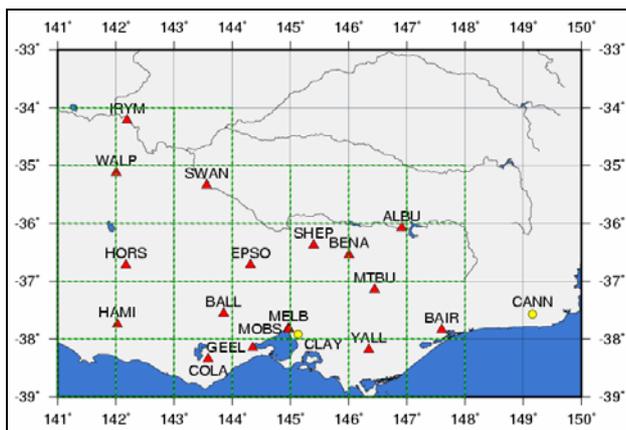


Fig. 1 Victoria GPS Network (GPSnet) site map

The Victoria's GPS Network – GPSnet as shown in Figure 1 is the network of permanent GPS base stations and supporting infrastructure that has been established co-operatively with a range of Industry partners, hosts and contributors; facilitated and operated by Land Victoria. The network records, distributes and archives GPS satellite correction data for accurate position determination, 24 hours a day, state-wide. GPSnet is

designed to international and national standards to meet the specific needs of Victorian GPS users. One can refer to GPSnet official website (<http://www.land.vic.gov.au>) for the current details of base station and operational information.

2.2 Data Collection and Processing

Since the zenith hydrostatic delay can be modelled and removed with an accuracy of a few millimetres or even better using a surface troposphere model, the residual tropospheric delay remaining after applying a standard model is mostly due to the wet component. Therefore, in order to have better prospect of the improvement of ZTD estimation by applying interpolated residual ZTDs, we intentionally chose a recent week from 8th to 14th September, 2004, during which the Victoria state was experiencing an extensive rainfall with the maximum weekly amounts up to 150 mm, as shown in Figure 2.

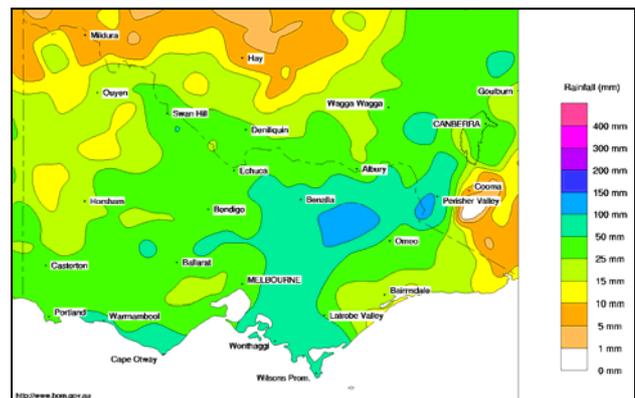


Fig. 2 One week rainfall amounts of Victorian region, from 8th to 14th September 2004, product of the National Climate Centre, Australia, retrieved from www.bom.gov.au

There are currently 19 base stations within the whole GPSnet. GPS raw measurements from 17 stations, as marked by red triangle in Figure 1, were collected and processed. The other two stations, CANN and CLAY, marked as yellow circle in Figure 1, were excluded from our analysis because of lack of data during the week we chose. The coordinates of all 17 stations of the network were precisely determined and tightly constrained to 5 mm, 5 mm, 10 mm in north, east, and up components, respectively. The corresponding IGS SP3 final orbits were also retrieved and held fixed during the processing.

GAMIT software (King et al., 2000) was used to process the data, which parameterizes ZTD as a stochastic variation from the Saastamoinen model. The variation is constrained to be a Gauss-Markov process with a power density of $2 \text{ cm}/\sqrt{h}$. ZTD estimates at 15-minute interval were solved with piecewise linear interpolation (PWL). A sliding window processing strategy (Fang et al., 1998) with a 12-hour window length and a 4-hour

forward step was used. However, the Gauss-Markov process provides an implicit constraint on the ZTD estimate at a given epoch from observations at preceding and following epochs, which means that the accuracy is expected to be lower at the beginning and end of each window. Therefore, to avoid the border effects of the Gauss-Markov filter, the central 4-hour ZTD estimates of each 12-hour session were extracted before moving the window forward.

3 Interpolating Residual Zenith Tropospheric Delays

3.1 Tropospheric Delay Model

To obtain the residual ZTDs, the modeled ZTD values need to be calculated first, and then removed from the GPS-derived ZTD estimates. The empirical tropospheric delay model used in our research is the Saastamoinen (SAAS) model (Saastamoinen, 1973), which reads:

$$\begin{aligned}
 ZTD &= ZHD + ZWD \\
 ZHD &= 0.0022767 \frac{P}{F(\varphi, H)} \\
 ZWD &= 0.0022767 \frac{e}{F(\varphi, H)} \left(\frac{1225}{t + 273.15} + 0.05 \right) \quad (1) \\
 F(\varphi, H) &= 1 - 0.00266 \cdot \cos 2\varphi - 0.00000028 \cdot H \\
 e &= RH \cdot 6.11 \cdot 10^{\frac{7.5t}{t+237.3}}
 \end{aligned}$$

where ZTD, ZHD and ZWD are zenith tropospheric delay, zenith hydrostatic (dry) delay and zenith wet delay, in m, respectively; φ is the ellipsoidal latitude of the station, H is station height in m; P is surface pressure in mbar, t is temperature in degree Celsius and RH is relative humidity in %, all at station height level.

If the meteorological parameters are given in reference to mean sea level (MSL), we need to convert them to the station level (SL), using the following relationships (Klein Baltink et al., 1999):

$$\begin{aligned}
 P_{SL} &= P_{MSL} \left(1 - 2.26 \times 10^{-5} \cdot H \right)^{5.225} \\
 T_{SL} &= T_{MSL} - 0.0065 \cdot H \\
 RH_{SL} &= RH_{MSL} \cdot \exp \left(-6.396 \times 10^{-4} \cdot H \right) \quad (2)
 \end{aligned}$$

Since no real-time meteorological data were applied to estimate ZTD using the SAAS model in our research, the standard values of surface pressure of 1013.25 mbar, temperature of 18°C, and relative humidity of 50%, at mean sea level (MSL), were used to calculate ZTD for all stations and all epochs. To consider the effect of height, we applied Equation (2) to convert the standard meteorological data at MSL to station level for every station.

3.2 Ordinary Kriging Interpolation

Ordinary Kriging (OK) is an interpolation procedure used in geostatistics, using known values in the neighbourhood and a variogram to determine the unknown values of the location being estimated. The variogram is based on spheroidal distance between points. A maximum range value a can be used to limit the distance that vanishes the covariance. Three kinds of variograms are suggested by Wackernagel (1998) to calculate the weight:

$$\text{Linear Variogram: } \gamma_{ij} = \begin{cases} \frac{d_{ij}}{a}, & \text{for } a \leq d_{ij} \\ 1, & \text{for } a > d_{ij} \end{cases}$$

$$\text{Exponential Variogram: } \gamma_{ij} = 1 - \exp(-d_{ij}/a)$$

$$\text{Spherical Variogram: } \gamma_{ij} = \begin{cases} \frac{3}{2} \frac{d_{ij}}{a} - \frac{1}{2} \frac{d_{ij}^3}{a^3}, & \text{for } a \leq d_{ij} \\ 1, & \text{for } a > d_{ij} \end{cases}$$

After the variogram is calculated, the weights of the sample points can be obtained as follows:

$$\omega = \mathbf{A}^{-1} \mathbf{B}; \quad \mathbf{A} = \begin{bmatrix} \gamma_{11} & \gamma_{21} & \cdots & \gamma_{n1} & 1 \\ \gamma_{12} & \gamma_{22} & \cdots & \gamma_{n2} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \cdots & \gamma_{nn} & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} \gamma_{1i} \\ \gamma_{2i} \\ \vdots \\ \gamma_{ni} \\ 1 \end{bmatrix}$$

With the weights determined, the value at point i can be calculated from the known values \mathbf{K} at the sample points $V_i = \omega \mathbf{K}$.

3.3 Tropospheric Grid Model User Algorithm

To obtain the residual ZTD corrections from the correction grid at a user location, a user's algorithm similar to the WAAS ionospheric grid model (Chao, 1997) was adopted. Figure 3 gives the user interpolation algorithm description. At each user, the residual tropospheric correction (T_u) is interpolated from the four grid points, so-called Tropospheric Grid Points (TGPs), surrounding that user:

$$T_u = \sum_{i=1}^4 \omega_i \cdot T_i \quad (3)$$

The weights (ω_i) in the interpolation are functions of TGP location and are calculated as recommended by WAAS MOPS Appendix A (RTCA SC-159, 1999). The general weighting function is:

$$\omega(x, y) = x^2 y^2 (9 - 6x - 6y + 4xy),$$

$$\text{for } 0 \leq x \leq 1, 0 \leq y \leq 1 \quad (4)$$

The x and y parameters are calculated from

$$x = \frac{\Delta\lambda_{IPP}}{\text{longitude grid interval}} \quad (5)$$

$$y = \frac{\Delta\phi_{IPP}}{\text{latitude grid interval}}$$

So, the weight of each corner T_u is:

$$\omega_1(x, y) = \omega(x, y) \quad \omega_2(x, y) = \omega(1 - x, y)$$

$$\omega_3(x, y) = \omega(1 - x, 1 - y) \quad \omega_4(x, y) = \omega(x, 1 - y)$$

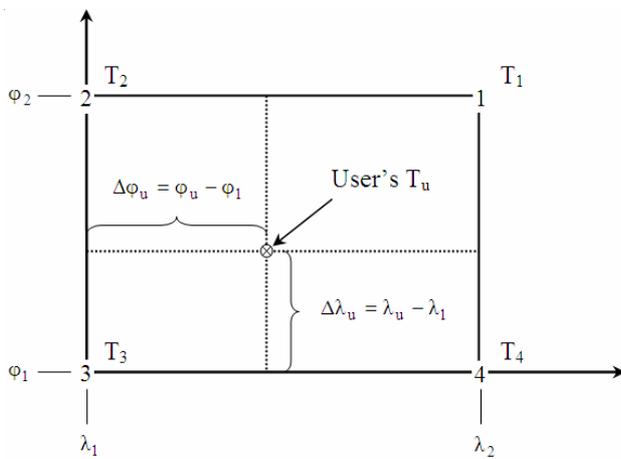


Fig. 3 User interpolation algorithm definition (Chao, 1997)

4 Results and Analysis

First of all, the error of the SAAS model itself against the GPS-derived ZTDs was examined to provide a criterion of the improvement after applying the interpolated residual corrections.

As mentioned before, for each station, identical meteorological data were used and converted from mean sea level to the station level to calculate ZTDs using the SAAS model. Therefore, the computed values for each station remain constant (straight line) during the whole week. Figure 4 illustrates the residuals of SAAS ZTD values with respect to GPS ZTD estimates for the station BAIR with the smallest difference, while Figure 5 illustrates the residuals for the station MTBU, having the largest errors. These residuals are considered as “true error” of the SAAS model at these stations with the given standard meteorological parameters.

After subtracting the SAAS modelled ZTD values from the GPS ZTD estimates, the residual ZTDs can be obtained and then interpolated. Two strategies are employed to compare and access the performance of OK interpolation. The first one is the identical strategy applied in our previous research, in which the residual ZTD of certain station at certain epoch is directly interpolated by OK method using the residuals of all other stations at the same epoch (as the block diagram enclosed by dash-line in Figure 6). The interpolated residual ZTDs obtained by this direct interpolation strategy are so-called “direct residual corrections”.

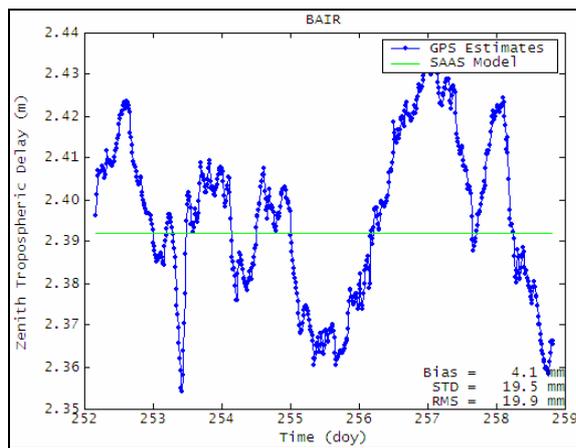


Fig. 4 SAAS ZTD values plotted against GPS ZTD estimates for the station BAIR

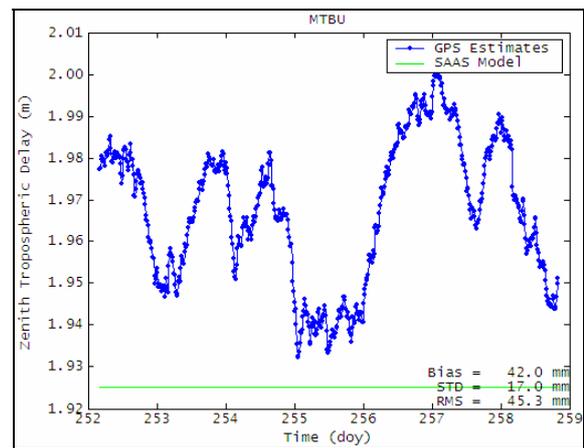


Fig. 5 SAAS ZTD values plotted against GPS ZTD estimates for the station MTBU

This direct strategy is suitable for error analysis, but as discussed, users will use a simple interpolation model to compute the correction from four grid points, instead of all the points of the network. To reflect this situation, the grid strategy is used. The grid strategy generates the corrections for each tested site using residual ZTDs of the network stations excluding the testing station through OK

interpolation, and then interpolates the residual ZTD of the excluded station from the grid using the user interpolation algorithm, as depicted in the block diagram enclosed by solid-line in Figure 6. The interpolated residual ZTDs obtained by this strategy may be called “grid residual ZTD corrections”. Within the geographic

span of GPSnet, a $1^\circ \times 1^\circ$ correction grid is created, as the green dash-line squares shown in Figure 1.

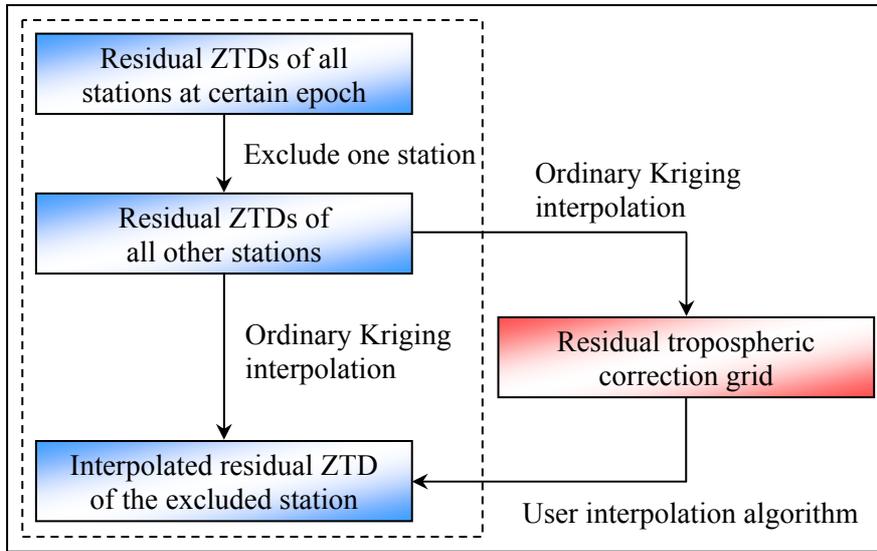


Fig. 6 Block diagram of residual zenith tropospheric delay interpolation strategies. The part enclosed by dash-line is the strategy of generating direct residual corrections, while the whole part enclosed by solid-line is the strategy of generating grid residual corrections.

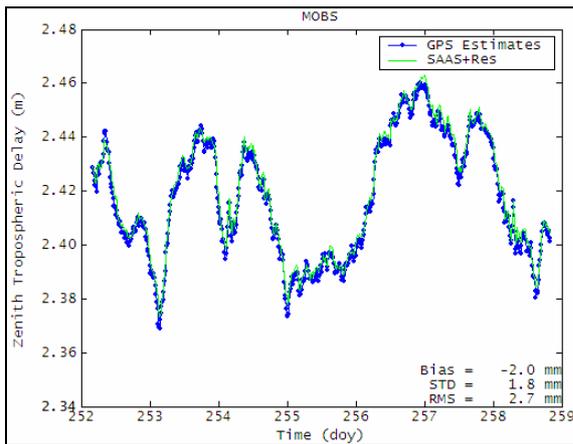


Fig.7 Comparison of GPS ZTD estimates and SAAS ZTD values with direct residual corrections for the station MOBS

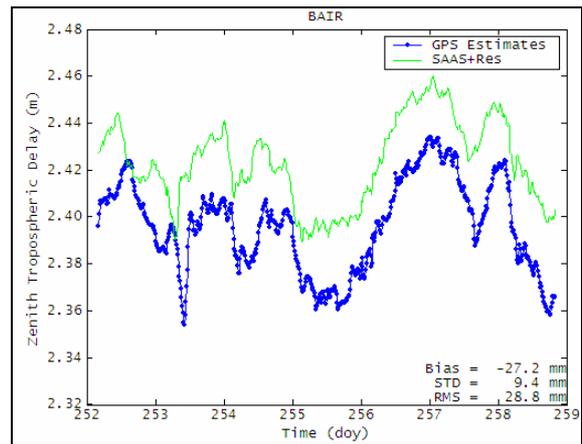


Fig.8 Comparison of GPS ZTD estimates and SAAS ZTD values with direct residual corrections for the station BAIR

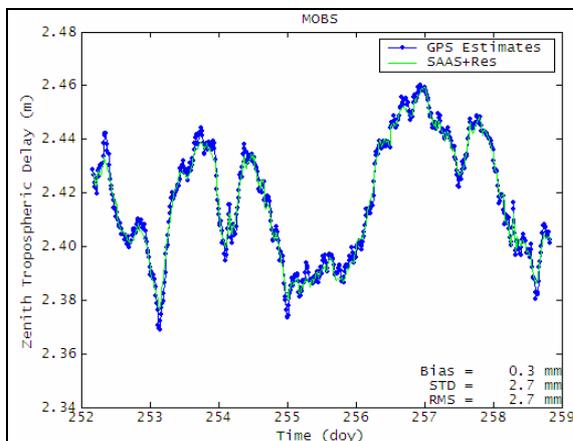


Fig.9 Comparison of GPS ZTD estimates and SAAS ZTD values with grid residual corrections for the station MOBS

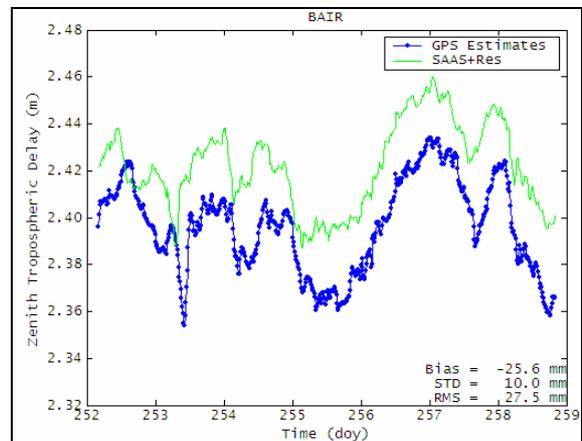


Fig.10 Comparison of GPS ZTD estimates and SAAS ZTD values with grid residual corrections for the station BAIR

Figures 7 and 8 plot the GPS-derived ZTDs against SAAS modelled ZTDs corrected with the direct residual corrections (SAAS+Res) for two different stations, MOBS and BAIR, where the best and the worst

consistencies were found, respectively. Figures 9 and 10 present the similar results for the SAAS model with the grid residual corrections (SAAS+Grid Res) for the same stations, MOBS and BAIR, respectively.

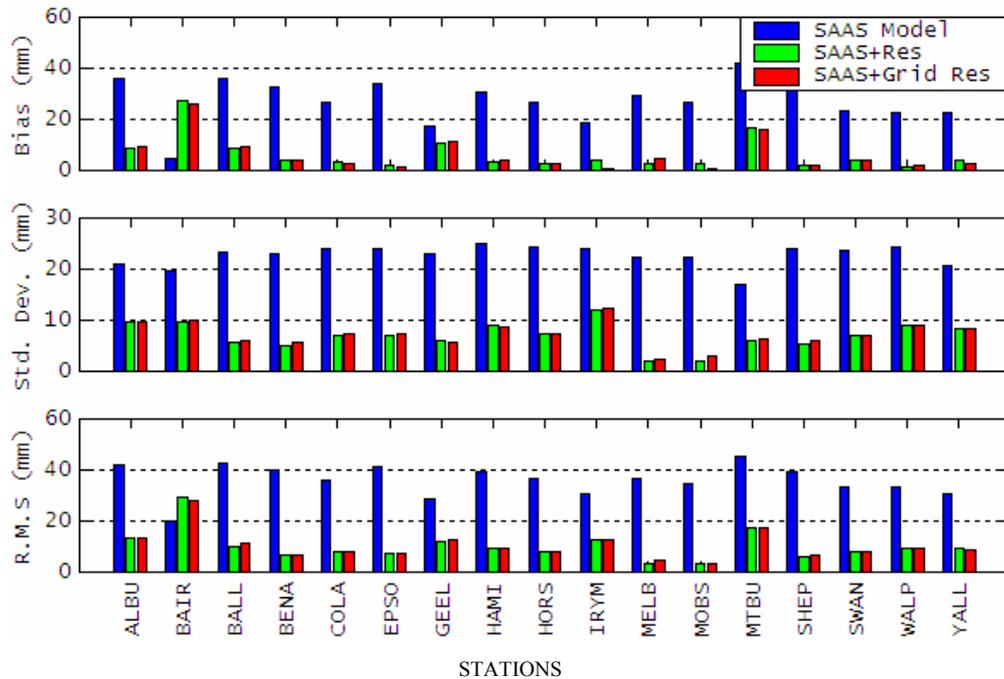


Fig. 11 Statistical comparisons of modeled ZTDs before corrections (SAAS model), after direct corrections (SAAS+Res), and after grid corrections (SAAS+Grid Res) with respect to GPS derived estimates for all stations. The upper figure shows the *absolute values* of biases, while the middle and bottom figures plot the standard deviation (STD) and root-mean-square (RMS) values, respectively.

Tab. 1 Overall statistics of SAAS ZTDs with respect to GPS estimates before corrections, after direct residual corrections, and after grid residual corrections (units in mm)

	SAAS	SAAS+Res	SAAS+Grid Res
Bias	26.8	-0.5	0.0
STD	24.2	11.4	11.4
RMS	36.1	11.4	11.4

Figures 11 compares the statistics for all 17 stations, showing the significant improvements by applying direct and grid residual corrections to the SAAS model values, respectively. Table 1 compares the overall bias, STD and RMS values before and after applying both interpolated residual corrections, demonstrating significant overall improvements after corrections. The biases are improved from 27 mm to zero, while the STDs are improved from 24 mm to 11 mm for both kinds of residual corrections. As it can be seen from the figures and the table, both interpolation strategies produce similar results of the interpolated corrections. Moreover, the overall statistic results of the grid residual corrections are even slightly better than the direct ones, which means the proposed tropospheric grid model and the user interpolation algorithm can be used to generate residual tropospheric corrections for near-real time or real-time applications

without accuracy degradation from grid interpolation.

It is also interesting to notice that at station BAIR, the bias value is the smallest among all the stations before interpolation but reaches the largest after interpolation, while the STD value has still been improved after interpolation. The reason of causing the large bias at station BAIR is still under closer investigation.

7 Conclusions

Considering spatial and temporal variability of the meteorological parameters, tropospheric delays in GPS differential positioning are usually corrected locally at the user end with an empirical model and a mapping function. In this paper, we have evaluated an approach to

providing additional tropospheric delay corrections for improved differential positioning.

One week GPS measurements of the GPSnet were processed and precise ZTD estimates were estimated every 15 minutes. After subtracting SAAS modeled values from GPS estimates, the residual ZTDs were obtained and then interpolated using Ordinary Kriging method by two approaches. One is "direct interpolation", and the other is "grid interpolation". However, slight differences were found between the results obtained from two approaches. After correcting with interpolated residual ZTDs, the bias caused by SAAS model is significantly improved from 27 mm to nearly zero, while the STD is improved from 24 mm to 11 mm for the tested data sets. Therefore, we can generally conclude that the RMS ZTD accuracy of about 1 cm is achievable over the Land Victoria GPS network coverage, using the proposed grid tropospheric correction strategy, which can support centimetre level positioning in the regional.

Finally, we summarise the interpolation procedures as follows: 1) reference stations and users use the same tropospheric model with the standard meteorological parameters converted from the mean sea level to the station level, to compute nominated troposphere delays in zenith directions; 2) the reference stations can use OK method to interpolate residual ZTDs and generate a tropospheric grid model, then broadcast it to the users within the service coverage area; 3) A user interpolates its residual ZTD correction from the grid model using the user interpolation algorithm. In all the process, no spatially and temporally variation of surface meteorological data needs to be involved.

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