

Application of Nonlinear Smoothing to Integrated GPS/INS Navigation System

Jaewon Seo, Jang Gyu Lee

School of Electrical Engineering and Computer Science, Seoul National University, Seoul, Korea
e-mail: jwseo1@snu.ac.kr; jgl@snu.ac.kr; Tel: +822-880-7317; Fax: +822-878-8198

Chan Gook Park

School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Korea
e-mail: chanpark@snu.ac.kr; Tel: +822-880-1675; Fax: +822-888-7958

Hyung Keun Lee

School of Electronis, Telecommunication, and Computer Engineering, Hankuk Aviation University, Gyeonggi-do, Korea
e-mail: hyknlee@hau.ac.kr; Tel: +822-300-0131; Fax: +822-3159-9257

Seong-Baek Kim

Team of Performance & Assessment, Korea Science and Engineering Foundation, Daejeon, Korea
e-mail: kimstar100@kosef.re.kr; Tel: + 8242-769-6321; Fax: + 8242-869-6314

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Abstract. The application of smoothing in the integrated navigation system of 4S-Van is considered. 4S-Van is a mobile mapping system, which provides the position information of various objects on the road. For navigation purpose, it has various sensors such as an inertial measurement unit, a GPS receiver and an odometer. It is also equipped with CCD cameras, laser scanners and video cameras, for mapping purpose. The navigation system of 4S-Van is based on an inertial navigation system, which is aided by GPS and the odometer. Because it can adopt a post-processing method for more accurate and reliable results, the nonlinear smoothing is applied. The nonlinear smoothing, which consists of a forward filter, a backward filter and a smoother, is implemented. For the forward filter, the extended Kalman filter is designed, and for the backward filter, the linearized Kalman filter is constituted. In the smoothing stage, the results of two filters are combined. The algorithm is applied to experimental data and the obtained result demonstrates the effectiveness and good performance of the nonlinear smoothing.

Key words: Inertial Navigation System, GPS, Nonlinear Smoother, Mobile Mapping System

1 Introduction

An Inertial Navigation System (INS) is a system that calculates the position, velocity, and attitude of a vehicle with the output of inertial sensors. The measurements of the inertial sensors contain errors due to physical limitations. These errors are accumulated in the navigation solution of INS, decreasing the accuracy of the solution. Therefore, if the error is not compensated with non-inertial sensors, the information of INS can only be trusted during a short period of time (Siouris, 1993; Titterton, 1997). Nowadays, most INSs are designed with aiding sensors such as GPS.

The mobile mapping system is an information system, which is used for constructing, maintaining, and managing the database of geographical information and facility circumstances. Compared with conventional method mostly depending on surveying, the mobile mapping system is more efficient and easy to operate. 4S-van in Fig. 1 is a mobile mapping system developed by Electronics and Telecommunications Research Institute (ETRI) in Korea. It utilizes CCD cameras, laser scanners, and video cameras for mapping. An inertial measurement unit, GPS receivers, and an odometer are integrated for navigation. With these sensors, the navigation result of 4S-van, such as location, velocity, and attitude can be calculated. For the high accuracy of the database

information, the navigation result must be accurate. Therefore, a reliable and accurate navigation system is indispensable for mobile mapping systems. Dissanayake *et al.* (2001) researched the land vehicle navigation system with an inertial measurement unit, a GPS receiver and an odometer. In this paper, for the GPS/INS integrated navigation system of 4S-van, the nonlinear smoother is implemented for efficient estimation of navigation errors and the indirect feedback method is designed for accurate calculation of the navigation solution. The nonlinearity of the navigation system model makes the nonlinear estimators inevitable.

The smoother is a non-real time estimator, which uses all measurements between 0 and T to estimate the state of a system at a certain time t_k , where $0 \leq t_k \leq T$. In general, smoothing is classified into three categories; fixed-interval, fixed-point, fixed-lag smoothing (Brown and Hwang, 1997). In the fixed-interval smoothing, the time interval of measurements is fixed and optimal estimates at some interior point is searched. Originally the smoother was proposed with three components: a forward filter, a separate backward filter, and a separate smoother (Gelb, 1974). Rauch *et al.* (1965) proposed a scheme with two components: a forward filter and backward smoothing algorithm which incorporates both the backward filter and the smoother. Fraser and Potter (1969) presented a solution to the smoothing problem by interpreting the optimum linear smoother as a combination of two optimum linear filters. For a nonlinear system, Leondes *et al.* (1970) proposed a nonlinear smoother and Yu *et al.* (2002) proposed a nonlinear smoother which is suitable for navigation systems. In this paper, the smoother proposed by Yu *et al.* (2002) is implemented for 4S-van.

The current paper is organized as follows. In section 2, the fixed-interval nonlinear smoother suitable for the navigation of 4S-van is explained. The navigation algorithm for 4S-van is described in section 3 and in section 4 the experimental result is provided. Finally, in section 5, the concluding remarks are presented.

2 Nonlinear smoother

Smoothing is an estimation method using the measurements before and after time t_k for the estimates of the states at time t_k . The following equations show the difference of filtering and smoothing.

$$\text{Filtering: } \hat{x}(t_k) = E\{x(t_k) | Z_k\} \quad (1)$$

$$\text{Smoothing: } \hat{x}(t_k) = E\{x(t_k) | Z_m\}, m > k \quad (2)$$

where $Z_k = \{z_1, z_2, \dots, z_k\}$ and it denotes a measurement set from t_1 to t_k . In equations (1) and (2), it is obvious that the smoothing cannot be used for real time applications.

Smoothers are classified into fixed-point, fixed-lag, and fixed-interval smoother (Brown and Hwang, 1997; Gelb, 1974). In a fixed-interval smoother, the time interval of measurements is fixed and optimal estimates at some interior point is sought. This is the typical problem encountered when processing noisy measurements in non-real time applications. The fixed-interval smoothers have been derived in a few different approaches (Brown and Hwang, 1997; Lewis, 1986). In this paper, however, one kind of the fixed-interval smoothers which is most efficient for the mobile mapping systems will be introduced. The smoother is basically constituted with a forward filter, an independent backward filter and a smoother. Forward and backward filters use measurements independently and the smoother fuses the results of the filters. In the subsequent sections, each filter and smoother will be explained.

The nonlinear system is considered as follows.

$$\dot{x}(t) = f(x(t), t) + G(t)w(t) \quad (3)$$

$$z(t_k) = h(x(t_k), t_k) + v(t_k) \quad (4)$$

where $w(t)$ is white Gaussian noise with zero mean and covariance $Q(t)$; $v(t_i)$ is also white Gaussian noise with zero mean and covariance $R(t)$.

2.1 Forward filter: Extended Kalman filter

As a forward filter of a fixed-interval nonlinear smoother, the extended Kalman filter is used. Assuming that the initial state has a normal distribution with mean \hat{x}_0 and covariance P_0 , the time propagation of the extended Kalman filter for the system given by equations (3) and (4), is expressed in the following equations:

$$\begin{aligned} \forall t_{k-1} \leq t < t_k \\ \dot{\hat{x}}(t | t_{k-1}) &= f(\hat{x}(t | t_{k-1}), t), \quad \hat{x}(t_{k-1} | t_{k-1}) = \hat{x}(t_{k-1}^+) \\ \dot{P}(t | t_{k-1}) &= F[t; \hat{x}(t | t_{k-1})]P(t | t_{k-1}) \\ &+ P(t | t_{k-1})F^T[t; \hat{x}(t | t_{k-1})] + G(t)Q(t)G^T(t), \\ P(t_{k-1} | t_{k-1}) &= P(t_{k-1}^+) \end{aligned} \quad (5)$$

where $F[t; \hat{x}(t | t_k)] = \left. \frac{\partial f(x(t), t)}{\partial x} \right|_{x=\hat{x}(t|t_k)}$. The notation $s(t | t_k)$ means that the value $s(t)$ is calculated

depending on the measurements obtained before and at time t_k .

At the time t_k when a new measurement is available, the filter updates the estimates with the information of the measurement:

$$\begin{aligned}\hat{x}(t_k^+) &= \hat{x}(t_k^-) + K_k \{z(t_k) - h(\hat{x}(t_k^-), t_k)\} \\ K_k &= P(t_k^-) H^T [t_k; \hat{x}(t_k^-)] \\ &\cdot \{H[t_k; \hat{x}(t_k^-)] P(t_k^-) H^T [t_k; \hat{x}(t_k^-)] + R(t_k)\}^{-1} \\ P(t_k^+) &= \{I - K_k H[t_k; \hat{x}(t_k^-)]\} P(t_k^-) \\ &\cdot \{I - K_k H[t_k; \hat{x}(t_k^-)]\}^T + K_k R(t_k) K_k^T \\ \hat{x}(t_k^-) &= \hat{x}(t_k | t_{k-1}), P(t_k^-) = P(t_k | t_{k-1})\end{aligned}\quad (6)$$

$$\text{where } H[t; \hat{x}(t_k^-)] = \left. \frac{\partial h(x(t_k), t_k)}{\partial x} \right|_{x=\hat{x}(t_k^-)}$$

An extended Kalman filter is used for online state estimation and considered as a modification of the linear Kalman filter for nonlinear systems. In the navigation system, the extended Kalman filter estimates the navigation solutions through the perturbed error model.

2.2 Backward filter: Linearized Kalman filter

In nonlinear systems, if the nominal trajectory for linearization is given, the linearized Kalman filter can be used for state estimation. In this paper, the nonlinear smoother adopts the linearized Kalman filter for the backward filter with some modifications (Yu, 2002).

Before explaining the linearized Kalman filter, the nominal trajectory is assumed to be given as follows:

$$\dot{x}_n(t) = f(x_n(t), t), x_n(t_0) = x_0. \quad (7)$$

The time propagation and measurement update of the filter is given as the following equations:

$$\begin{aligned}\forall t_{k-1} \leq t < t_k \\ \dot{\hat{x}}(t | t_{k-1}) &= f(x_n(t), t) \\ &+ F[t; x_n(t)] \{\hat{x}(t | t_{k-1}) - x_n(t)\} \\ \dot{P}(t | t_{k-1}) &= F[t; x_n(t)] P(t | t_{k-1}) \\ &+ P(t | t_{k-1}) F^T [t; x_n(t)] + G(t) Q(t) G^T (t) \\ \hat{x}(t_{k-1} | t_{k-1}) &= \hat{x}(t_{k-1}^+), P(t_{k-1} | t_{k-1}) = P(t_{k-1}^+)\end{aligned}\quad (8)$$

$$\begin{aligned}\hat{x}(t_k^+) &= \hat{x}(t_k^-) + K_k \{z(t_k) - h(x_n(t_k), t_k) \\ &- H[t_k; x_n(t)] \{\hat{x}(t_k^-) - x_n(t_k)\}\} \\ K_k &= P(t_k^-) H^T [t_k; x_n(t)] \\ &\cdot \{H[t_k; x_n(t)] P(t_k^-) H^T [t_k; x_n(t)] + R(t_k)\}^{-1} \\ P(t_k^+) &= \{I - K_k H[t_k; x_n(t)]\} P(t_k^-) \\ &\cdot \{I - K_k H[t_k; x_n(t)]\}^T + K_k R(t_k) K_k^T \\ \hat{x}(t_k^-) &= x(t_k | t_{k-1}), P(t_k^-) = P(t_k | t_{k-1})\end{aligned}\quad (9)$$

$$\text{where } F[t; x_n(t)] = \left. \frac{\partial f(x(t), t)}{\partial x} \right|_{x=x_n(t)}$$

$$\left. \frac{\partial h(x(t_k), t_k)}{\partial x} \right|_{x=x_n(t_k)}$$

For the backward implementation of the linearized Kalman filter, the variable $\tau = t_f - t$ and equations (10) are newly defined. t_f is the final time in the fixed time interval of the smoothing.

$$\begin{aligned}\hat{y}_b(t_f^-) &= P_b^{-1}(t_f^-) \hat{x}_b(t_f^-) = 0 \\ \hat{y}_b(t_k^-) &= P_b^{-1}(t_k^-) \hat{x}_b(t_k^-) \\ \hat{y}_b(t_k^+) &= P_b^{-1}(t_k^+) \hat{x}_b(t_k^+)\end{aligned}\quad (10)$$

where the subscript b denotes that the variables are of the backward filter. Because the backward filter has no information about initial states at time t_f in most cases,

$P_b(t_f^-) = \infty$ or $P_b^{-1} = 0$ is naturally assumed. Therefore, it is converted into an information filter with definitions (10) and $\tau = t_f - t$ for the convenience of the implementation. The derived backward filter is given in (11).

$$\begin{aligned}\frac{d}{d\tau} P_b^{-1}(\tau) &= P_b^{-1}(\tau) F[t_f - \tau; \hat{x}_f(t_f - \tau)] \\ &+ F^T [t_f - \tau; \hat{x}_f(t_f - \tau)] P_b^{-1}(\tau) \\ &- P_b^{-1}(\tau) G(t_f - \tau) Q(t_f - \tau) G^T (t_f - \tau) P_b^{-1}(\tau) \\ \frac{d}{d\tau} \hat{y}_b(\tau) &= \{F^T [t_f - \tau; \hat{x}_f(t_f - \tau)] \\ &- P_b^{-1}(\tau) G(t_f - \tau) Q(t_f - \tau) G^T (t_f - \tau)\} \hat{y}_b(\tau) \\ &- P_b^{-1}(\tau) \{f(\hat{x}_f(t_f - \tau), t_f - \tau) \\ &- F[t_f - \tau; \hat{x}_f(t_f - \tau)] \hat{x}_f(t_f - \tau)\} \\ \hat{y}_b(0) &= 0, P_b^{-1}(0) = 0 \\ \hat{y}_b(\tau_k^+) &= \hat{y}_b(\tau_k^-) + H^T (\tau_k) R^{-1}(\tau_k) \{z(\tau_k) \\ &- h(\hat{x}_f(\tau_k^-), \tau_k) + H(\tau_k) \hat{x}_f(\tau_k^-)\}\end{aligned}\quad (11)$$

$$P_b^{-1}(\tau_k^+) = P_b^{-1}(\tau_k^-) + H^T(\tau_k)R^{-1}(\tau_k)H(\tau_k)$$

where $\hat{x}_f(\cdot)$ is the result of the forward filter and in this smoother, it is used for the nominal trajectory of the linearized Kalman filter.

For construction of the indirect feedback navigation system, the backward filter (11) should be perturbed and the following equations can be obtained.

$$\begin{aligned} \delta \dot{\hat{y}}_b(\tau) &= \{F^T[t_f - \tau; \hat{x}_f(t_f - \tau)] \\ &- P_b^{-1}(\tau)G(t_f - \tau)Q(t_f - \tau)G^T(t_f - \tau)\} \delta \hat{y}_b(\tau) \\ \dot{P}_b^{-1}(\tau) &= P_b^{-1}(\tau)F[t_f - \tau; \hat{x}_f(t_f - \tau)] \\ &+ F^T[t_f - \tau; \hat{x}_f(t_f - \tau)]P_b^{-1}(\tau) \\ &- P_b^{-1}(\tau)G(t_f - \tau)Q(t_f - \tau)G^T(t_f - \tau)P_b^{-1}(\tau) \end{aligned} \quad (12)$$

$$\begin{aligned} \delta \hat{y}_b(\tau_k^+) &= P_b^{-1}(\tau_k^-) \delta \hat{x}_f(t_f - \tau_k^+) \\ &+ \delta \hat{y}_b(\tau_k^-) - H(\tau_k)R^{-1}(\tau_k) \delta z(\tau_k) \\ P_b^{-1}(\tau_k^+) &= P_b^{-1}(\tau_k^-) + H^T(\tau_k)R^{-1}(\tau_k)H(\tau_k) \end{aligned} \quad (13)$$

where $\delta \hat{y}_b(0) = 0$ and $P_b^{-1}(0) = 0$ (Yu, 2002).

2.3 Smoother

The estimated states $\hat{x}_s(t_k)$ of the smoother are calculated with the forward filter result, $\hat{x}_f(t_k)$ and the backward filter result, $\delta \hat{y}_b(\tau_k)$.

$$\begin{aligned} \hat{x}_s(t_k) &= \hat{x}_f(t_k^+) - P_s(t_k) \delta \hat{y}_b(\tau_k^-) \\ P_s^{-1}(t_k) &= P_f^{-1}(t_k^+) + P_b^{-1}(\tau_k^-) \end{aligned} \quad (14)$$

3 Navigation algorithm

For 4S-van navigation, the indirect feedback method is applied. In the indirect feedback navigation, the navigation errors are estimated with some estimator, and then the navigation solutions are compensated with the estimated errors (Titterton, 1997). In the case of 4S-van, the measurements of GPS and an odometer are used as aiding information for the inertial navigation system.

For application of nonlinear smoothing to the navigation system of 4S-van, the inertial navigation system error model will be necessary. The inertial navigation system error model is described as follows:

$$\delta \dot{L} = \frac{\rho_E}{R_m + h} \delta h + \frac{1}{R_m + h} \delta v_N$$

$$\delta \dot{l} = \rho_N \sec L \tan L \delta L - \frac{\rho_N \sec L}{R_t + h} \delta h + \frac{\sec L}{R_t + h} \delta v_E$$

$$\begin{aligned} \delta \dot{h} &= -\delta v_D \\ \delta \dot{v}^n &= [C_b^n f_b] \times \phi - [2\omega_{ie}^n + \omega_{en}^n] \times \delta v^n \\ &+ C_b^n \delta f^b + v^n \times (2\delta\omega_{ie}^n + \delta\omega_{en}^n) \\ \phi &= -\omega_{in}^n \times \phi - C_b^n \delta\omega_{ib}^b + \delta\omega_{in}^n \end{aligned} \quad (15)$$

where $\delta L, \delta l, \delta h, \delta v^n$ and ϕ are the errors of latitude, longitude, height, velocity in the navigation frame, and small tilt angle, respectively. R_m, R_t and C_b^n are the meridian radius, transverse radius, and transformation matrix from the body frame to the navigation frame, respectively. $\rho_{N,E,D}, \delta\omega_{ie}^n$ and $\delta\omega_{en}^n$ are defined as follows:

$$\begin{aligned} [\rho_N, \rho_E, \rho_D]^T &= [l \cos L, -L, -l \sin L]^T \\ \delta\omega_{ie}^n &= [-\Omega \sin L \delta L, 0, -\Omega \cos L \delta L]^T \\ \delta\omega_{ie}^n &\approx \begin{bmatrix} -\frac{\rho_N}{R_t + h} \delta h + \frac{1}{R_t + h} \delta v_E \\ -\frac{\rho_E}{R_m + h} \delta h + \frac{1}{R_m + h} \delta v_N \\ \rho_N \sec^2 L \delta L - \frac{\rho_D}{R_t + h} \delta h + \frac{\rho_D}{v_E} \delta v_E \end{bmatrix} \end{aligned} \quad (16)$$

$$\omega_{in}^n = \omega_{ie}^n + \omega_{en}^n, \delta\omega_{in}^n = \delta\omega_{ie}^n + \delta\omega_{en}^n$$

where Ω is the earth rate. δf^b is the accelerometer noise and $\delta\omega_{ib}^b$ is the gyroscope noise. These noises are modelled as follows:

$$\begin{aligned} \delta f^b &= \nabla_a + \omega_a \\ \delta\omega_{ib}^b &= \varepsilon_g + \omega_g \\ \dot{\nabla}_a &= 0 \\ \dot{\varepsilon}_g &= 0 \end{aligned} \quad (17)$$

where ∇_a and ε_g are the random bias noises; ω_a and ω_g are the white Gaussian noises. The odometer scale factor error must be compensated. The scale factor error, k_{xo} , is assumed to be a random constant with an initial distribution of $N(0, Q_K)$. The state vectors are given in (18).

$$\delta x^T = [\delta L \quad \delta l \quad \delta h \quad \delta v_N \quad \delta v_E \quad \delta v_D \quad \phi_N \quad \phi_E \quad \phi_D \quad \nabla_a \quad \varepsilon_g \quad k_{xo}] \quad (18)$$

The odometer observation model is given as follows:

$$V_m^b \approx (1 - k_{xo})V_x^b \quad (19)$$

where V_m^b is the output of the odometer, k_{xo} is the scale factor error and V_x^b is the forward direction velocity of the vehicle, respectively. From equation (19), the error model applied to the smoother is derived as follows (Yu, 1999):

$$\delta V_m = \hat{V}^n - V_m^n = \delta v^n - \hat{V}^n \times \begin{bmatrix} \phi_N \\ \phi_E \\ \phi_D \end{bmatrix} - \hat{C}_b^n V_x^b \Delta k + v_o, \quad (20)$$

$$\Delta k = k_{xo} - \hat{k}_{xo}$$

where \hat{V}^n is the velocity estimates in the navigation frame, V_m^n is the measured velocity by the odometer in the navigation frame, \hat{C}_b^n is the transformation matrix calculated with the estimated values, \hat{k}_{xo} is the estimated scale factor error and v_o is the white Gaussian noise with zero mean and appropriate covariance.

Position measurement of a GPS receiver is modelled as follows:

$$\delta P = \hat{P} - P_m = \delta P + v_p \equiv \begin{bmatrix} \delta L \\ \delta l \\ \delta h \end{bmatrix} + v_p \quad (21)$$

where \hat{P} is the estimated position vector in latitude, longitude and height, P_m is the measured data and v_p is the white Gaussian noise with zero mean and appropriate covariance.

The covariances are given in the manufacturer's specification table or can be determined by the statistical result of repetitive experiments.

4 Experimental result

4S-van in Fig. 1 is a mobile mapping system which has been developed by the Electronics and Telecommunications Research Institute in Korea. It has CCD cameras, lase scanners, and video cameras for mapping. For constitution of integrated navigation system, the LN-200 developed by Northrop Grumman Co. is used as an inertial measurement unit and two GPS receivers by Trimble Co. are equipped on top of the vehicle. The position information for the aided inertial navigation system is acquired in the DGPS method (Hofmann-Wellenhof, 1994; Kaplan, 1996). Therefore, the provided position measurements are very accurate. The noise characteristics of the sensors are given by the

manufacturers and the dominating errors are given in Table 1.



Fig. 1 4S-van

Tab. 1 Specifications of sensors

Sensors		Standard deviation (1σ)	
IMU (LN-200)	Accelerometer	Bias repeatability	1mg
		Scale factor stability	300ppm
		Bias variation	50ug with 60 second correlation time
		White noise	50ug
		Bandwidth	100Hz
	Gyro	Bias repeatability	1° / hr
		Random walk	0.07° / \sqrt{hr}
		Scale factor stability	100ppm
		Bias variation	0.35° / hr with 100 second correlation time
		Bandwidth	>500Hz
DGPS (Position accuracy)		50cm	
Odometer (Velocity accuracy)		1.2m/s	

The test area in this experiment is an asphalt-paved road in Daejeon, Korea. Total distance of navigation is about 11km and total time is 2150 seconds. The heights above the mean sea level change from 50m to 110m in that area.

The navigation results of the experiment are shown in Fig. 2, Fig. 3 and Fig. 4. Fig. 2 shows the navigation result in plane view. Fig. 3 shows the results of north direction, east direction and height. Fig. 4 shows the estimated attitude result. The estimation errors of the result are summarized in Table 2.

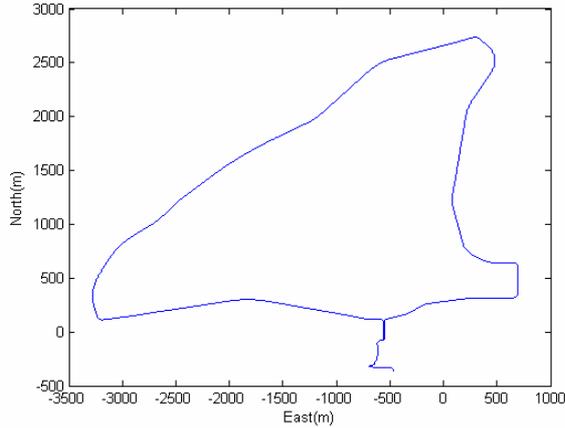


Fig. 2 The estimated position result of 4S-van in plane view

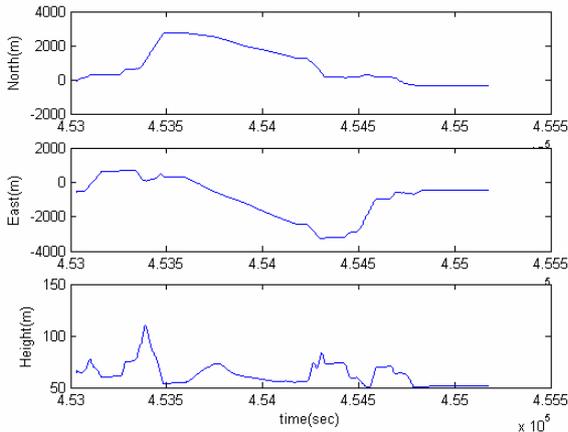


Fig. 3 The estimated position result in north, east and height direction

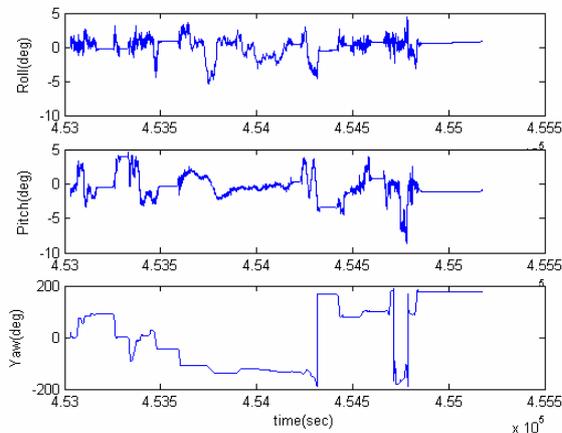


Fig. 4 The attitude estimation result of 4S-van

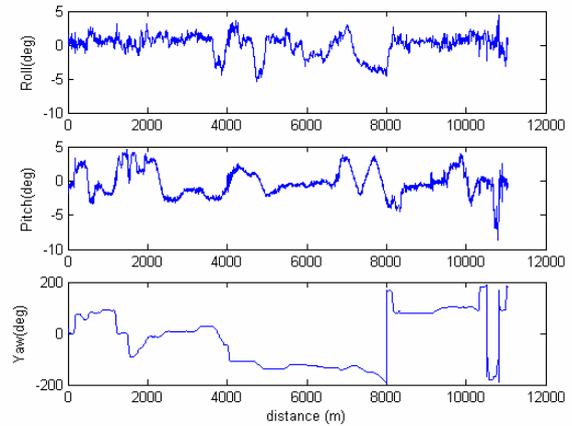


Fig. 4 The attitude estimation result of 4S-van (continued)

Tab. 2 The navigation errors

	Error (RMS)
North	0.299m
East	0.396m
Height	0.726m
Roll	0.248deg
Pitch	0.214deg
Yaw	0.201deg

According to these results, 4S-van can provide its position with errors of 30~40cm in the horizontal plane. The attitude error of 4S-van also causes the position error in the mapping result. For the case of a 30m-apart object from 4S-van, the error of 0.1 deg in the attitude of 4S-van corresponds to about 5cm error in the position of the object. Therefore, the above navigation results cause mapping errors up to about 60cm in horizontal plane which can be calculated with north, east, and yaw errors. With the current algorithm and 4S-van hardware, the map of about 60cm-errors can be made and it is sufficient for the intermediate level of mobile mapping.

5 Conclusion

In this paper, the nonlinear smoother is applied to the GPS/INS integrated navigation system of 4S-van. 4S-van is equipped with an inertial measurement unit, GPS receivers and an odometer for navigation. For accurate navigation solution, the error model of inertial navigation system is studied and the nonlinear smoothing algorithm is applied. The approximate error model is linear and 16th order. With this error model, the fixed-interval nonlinear smoothing is implemented. It consists of three components: Extended Kalman filter, Linearized Kalman filter, smoother. The implemented algorithm is applied to

the data obtained by an experiment. The result of the implemented algorithm has sufficient accuracy for mapping applications.

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References

- Brown R. G.; Hwang P. Y. C. (1997): *Introduction to random signals and applied Kalman filtering*, John Wiley & Sons, Inc.
- Dissanayake G.; Sukkarieh S.; Nebot E.; Durrant-Whyte H. (2001): The aiding of a low-cost strapdown inertial measurement unit using vehicle model constraints for land vehicle applications, *Institute of Electrical and Electronics Engineers Transactions on Robotics and Automation*, 17(5):731-747
- Fraser D. C.; Potter J. E. (1969): The optimum linear smoother as a combination of two optimum linear filters, *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, 7(8):387-390
- Gelb A. (1974): *Applied optimal estimation*, The MIT Press
- Hofmann-Wellenhof B.; Lichtenegger H.; Collins J. (1994): *Global positioning system: theory and practice*, Springer-Verlag Wien New York
- Kaplan E. D. (1996): *Understanding GPS principles and applications*, Artech House, Inc.
- Leondes C. T.; Peller J. B.; Stear E. B. (1970): Nonlinear smoothing theory, *Institute of Electrical and Electronics Engineers Transactions on System Science and Cybernetics*, 6(1):63-71
- Lewis F. L. (1986): *Optimal estimation*, John Wiley & Sons, Inc.
- Rauch H. E.; Tung F.; Striebel C. T. (1965): Maximum likelihood estimates of linear dynamic systems, *American Institute of Aeronautics and Astronautics Journal*, 3(8):1445-1450
- Siouris G. M. (1993): *Aerospace avionics systems*, Academic Press, Inc
- Titterton D. H.; Weston J. L. (1997): *Strapdown inertial navigation technology*, Peter Peregrinus Ltd., :385-386
- Yu M.-J.; Lee J. G.; Park H.-W. (1999): Comparison of SDINS in-flight alignment using equivalent error models, *Institute of Electrical and Electronics Engineers Transactions on Aerospace and Electronic Systems*, 35(3):1046-1054
- Yu J.; Lee J. G.; Park C. G.; Han H. S. (2002): Design of nonlinear fixed-interval smoother for off-line navigation, *Journal of Control, Automation and Systems Engineering*, 8(11):984-990