

Evaluating the Performances of Adaptive Kalman Filter Methods in GPS/INS Integration

Ali Almagbile, Jinling Wang, and Weidong Ding

School of surveying & Spatial Information Systems, University of New South Wales, Sydney, NSW 2052, Australia

Abstract

One of the most important tasks in integration of GPS/INS is to choose the realistic dynamic model covariance matrix Q and measurement noise covariance matrix R for use in the Kalman filter. The performance of the methods to estimate both of these matrices depends entirely on the minimization of dynamic and measurement update errors that lead the filter to converge. This paper evaluates the performances of adaptive Kalman filter methods with different adaptations. Innovation and residual based adaptive Kalman filters were employed for adapting R and Q . These methods were implemented in a loose GPS/INS integration system and tested using real data sets. Their performances have been evaluated and compared. Their limitations in real-life engineering applications are discussed.

Keywords: GPS/INS integration; Kalman filter; Adaptive Kalman filter

1. Introduction

The Kalman filter (KF) technique has been widely implemented for GPS/INS integration systems. Kalman filters rely on dynamic and stochastic models (e.g., Hu et al, 2003) that describe the behaviour of the state vector and the relationship between the measurements and the state vector respectively. The optimality of Kalman filter depends on the quality of prior assumptions about the process noise covariance matrix Q and the measurements noise covariance R (Mohamed and Schwarz, 1999). The quality of prior assumptions which are determined by certain knowledge about the measurements and test analysis are crucial factors that lead to the optimality of the Kalman filtering technique. For instance, inadequacy of prior assumptions to represent the real noise level could lead to unreliable results and sometimes to filter divergence (Ding et al, 2007).

An adaptive Kalman filter has been used to tune the measurement and process noise covariance matrices R

and Q respectively. Determining the suitable values of R and Q plays an important role to obtain a converged filter (Mohamed and Schwarz, 1999). For example, unreliable results will be yielded in case of determining small values of Q and R , on the other side, big diagonal element values of Q and R could produce filter divergence. Consequently, much attention has been paid to determine the disturbance matrices in order to obtain optimal Kalman filter parameters especially in GPS/INS integration applications (e.g., Mehra, 1970, 1971, 1972; Moghaddamjoo and Kirilin, 1989; Mohammad and Schwarz, 1999; Wang et al, 1999; Hide et al, 2003; Li and Wang, 2006; Ding et al, 2007).

Adaptive Kalman filters have been developed using three different scenarios of adaptation. These adaptation scenarios are: adapting dynamic noise covariance matrix Q , measurement noise covariance matrix R , and the initial values of the error covariance matrix P . One of the philosophies for the Kalman filtering adaptation is to fix P and Q and vary R by trial and error to find the smallest value that gives stable state estimates, if this design does not give satisfactory performance, P and Q should also be varied (Grooves, 2008). Various approaches have already been proposed for estimating Q and R matrices. Mehra (1972) categorized these approaches as: Bayesian, Maximum likelihood, correlation and covariance matching methods. All of these methods have been tested in different applications in order to achieve high performance of Kalman filtering. For instance, the maximum likelihood (ML) method was employed in adaptive Kalman filtering (Mohamed and Schwarz, 1999). It is noted that this method provides reliable results for the GPS/INS integration algorithm. However, this method as well as Bayesian method need intensive computation and both are based on the assumption that the dynamic error is time-invariant, which is not realistic (Wang, 1999).

In this paper, three innovation and residual based adaptive Kalman filtering techniques have been evaluated in a loosely coupled GPS/INS integrated system. A comparison has been conducted based on covariance analysis, and innovation and residual

analysis. Influence of moving window sizes on adaptive filtering performance has been investigated. The structure of this paper is as follows: in Section 2 conventional and adaptive Kalman filtering methods are introduced. Results and analysis forms Section 3 followed by the concluding remarks.

2. Adaptive Kalman Filtering

2.1 Conventional Kalman filter

The linear discrete Kalman filter for a state dynamic and measurement models can be expressed as follows:

$$\mathbf{x}_k = \Phi_{k-1}\mathbf{x}_{k-1} + \mathbf{w}_{k-1} \quad (1)$$

$$\mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k \quad (2)$$

\mathbf{x}_k is the $(n \times 1)$ state vector;

Φ_k is the $(n \times n)$ transition matrix;

\mathbf{z}_k is the $(r \times 1)$ observation vector;

\mathbf{H}_k is the $(r \times n)$ observation matrix;

\mathbf{w}_k and \mathbf{v}_k are uncorrelated white noise sequences with the following mean and covariance:

$$E\{\mathbf{w}_k\} = E\{\mathbf{v}_k\} = 0 \quad (3)$$

$$E\{\mathbf{w}_k\mathbf{v}_k^T\} = 0 \quad (4)$$

$$E\{\mathbf{w}_k\mathbf{w}_k^T\} = \begin{cases} \mathbf{Q}_k, (i=k) \\ 0, (i \neq k) \end{cases} \quad (5)$$

$$E\{\mathbf{v}_k\mathbf{v}_k^T\} = \begin{cases} \mathbf{R}_k, (i=k) \\ 0, (i \neq k) \end{cases} \quad (6)$$

$E\{\cdot\}$ denotes the expectation function;

\mathbf{Q} and \mathbf{R} are the covariance matrix of process noise and measurement errors respectively.

The Kalman filter state prediction and state covariance prediction are:

$$\bar{\mathbf{x}}_k = \Phi_{k-1}\hat{\mathbf{x}}_{k-1} \quad (7)$$

$$\bar{\mathbf{P}}_k = \Phi_{k-1}\hat{\mathbf{P}}_{k-1}\Phi_{k-1}^T + \mathbf{Q}_{k-1} \quad (8)$$

$\hat{\mathbf{x}}_k$ denotes the estimated state vector;

$\bar{\mathbf{x}}_k$ is the predicted state vector for the next epoch;

$\hat{\mathbf{P}}_k$ is the estimated state covariance matrix;

$\bar{\mathbf{P}}_k$ is the predicted state covariance matrix.

The Kalman filter update steps are as follows:

$$\mathbf{K}_k = \bar{\mathbf{P}}_k\mathbf{H}_k^T(\mathbf{H}_k\bar{\mathbf{P}}_k\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \quad (9)$$

$$\mathbf{v}_k = \mathbf{z}_k - \mathbf{H}_k\bar{\mathbf{x}}_k \quad (10)$$

$$\hat{\mathbf{x}}_k = \bar{\mathbf{x}}_k + \mathbf{K}_k\mathbf{v}_k \quad (11)$$

$$\hat{\mathbf{P}}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\bar{\mathbf{P}}_k \quad (12)$$

\mathbf{K}_k is the Kalman gain, which defines the updating weight between the new measurements and the prediction from the system dynamic model.

2.2 Adaptive estimation of covariance matrices \mathbf{Q} and \mathbf{R}

The covariance matrices \mathbf{Q} and \mathbf{R} could be estimated using Minimum Norm Quadratic Unbiased Estimation (MINQUE) which is not suitable for real-time kinematic positioning (Wang, 1999). The method has been employed to estimate such matrices since it has very well-defined properties. One of the drawbacks of this method is that it requires iterative procedures which depend on the properties of the data and the model themselves (Wang et al, 1999). Therefore, it is unsuitable for real-time kinematic positioning.

In online stochastic modelling, however adaptive Kalman filtering techniques can be adopted since they provide online estimation of dynamic and measurement noise covariance matrices \mathbf{Q} and \mathbf{R} respectively. One of the adaptive Kalman filtering techniques is covariance matching which makes the elements of the innovation or residual-based covariance matrix consistent with their theoretical values (Maybeck, 1982). The estimated covariance matrix of the innovations or residuals should match its theoretical form. The innovation \mathbf{v}_k which is the difference between the real observations and its predicted value can be computed as follows:

$$\mathbf{v}_k = \mathbf{z}_k - \mathbf{H}_k\bar{\mathbf{x}}_k \quad (13)$$

The residual $\bar{\mathbf{v}}_k$ which is the difference between the real observations and its estimated values can be expressed as:

$$\bar{\mathbf{v}}_k = \mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_k \quad (14)$$

Based on the above assumption, the philosophy of estimating \mathbf{Q} and \mathbf{R} matrices takes one of these following scenarios:

- a) Fixing \mathbf{Q} and varying \mathbf{R} by trial and error until the realistic values are found that give stable state

estimates. In this case the Q matrix should be completely known; or

- b) Varying Q matrix if R is completely known and fixed to the certain value, otherwise;
- c) Varying Q and R simultaneously, in this case none of them is known and the initial values for both matrices should be selected carefully in order to find the best stable estimate.

2.2.1 Adaptive estimation of R based on innovation sequences

Following the procedures proposed by (Mehra, 1970, 1971; Mohamed and Schwarz, 1999; Yang and Xu, 2003), the measurement noise covariance matrix R can be adapted based on the innovation sequences as:

$$\hat{\mathbf{R}}_k = \hat{\mathbf{C}}_v - \mathbf{H}_k \hat{\mathbf{P}}_k \mathbf{H}_k^T \quad (15)$$

$\hat{\mathbf{C}}_v$ is the estimated variance-covariance V - C matrix of the innovation and it can be computed through averaging inside a moving window of size m at epoch k (Mohamed and Schwarz, 1999):

$$\hat{\mathbf{C}}_v = \frac{1}{m} \sum_{i=1}^m \mathbf{v}_{k-i} \mathbf{v}_{k-i}^T \quad (16)$$

2.2.2 Adaptive estimation of R based on residual sequences

When implementing the innovation based estimation for R as in equation (15), the outcomes must be positive definite. However, this outcome is not guaranteed since two positive definite matrices are subtracted. Therefore, Wang et al. (2000) proposed the residual based estimation for R in order to get positive definite outcomes. In this case the R matrix takes the following form (Wang et al, 1999):

$$\hat{\mathbf{R}}_k = \hat{\mathbf{C}}_{\bar{v}} + \mathbf{H}_k \hat{\mathbf{P}}_k \mathbf{H}_k^T \quad (17)$$

where $\hat{\mathbf{C}}_{\bar{v}}$ is calculated as:

$$\hat{\mathbf{C}}_{\bar{v}} = \frac{1}{m} \sum_{i=1}^m \bar{\mathbf{v}}_{k-i} \bar{\mathbf{v}}_{k-i}^T \quad (18)$$

2.2.3 Adaptive estimation of Q

Estimation of the dynamic noise covariance matrix Q is linked with measurement noise covariance matrix R since estimation of R requires the predicted state covariance $\mathbf{P}_{\bar{k}}$ and hence Q . Based on covariance matching principles, R is estimated using innovation or residual series based on Equations (15) or (17), respectively.

If R and $\mathbf{P}_{\bar{k}}$ are assumed to be known, Q can be scaled through calculating the ratio between the estimated innovation covariance and the predicted one (Ding et al, 2007):

$$\alpha = \frac{\text{trace}\{\hat{\mathbf{C}}_v - \mathbf{R}_k\}}{\text{trace}\{\mathbf{H}_k \hat{\mathbf{P}}_k \mathbf{H}_k^T\}} \quad (19)$$

By substituting (8) into (19), the scaling factor α takes this form:

$$\alpha = \frac{\text{trace}\{\mathbf{H}_k (\Phi_{k-1} \hat{\mathbf{P}}_{k-1} \Phi_{k-1}^T + \tilde{\mathbf{Q}}_{k-1}) \mathbf{H}_k^T\}}{\text{trace}\{\mathbf{H}_k (\Phi_{k-1} \hat{\mathbf{P}}_{k-1} \Phi_{k-1}^T + \mathbf{Q}_{k-1}) \mathbf{H}_k^T\}} \quad (20)$$

Based on (19) and (20), Q at epoch k can be scaled as:

$$\hat{\mathbf{Q}}_k = \mathbf{Q}_{k-1} \sqrt{\alpha} \quad (21)$$

The scaling factor α in (21) can be within a range greater or less than one. This provides an increase of the probability of tuning Q . Nonetheless, the value of α can be equal to one when the predicted and estimated innovation covariance has the same value. Details of Q derivation can be found in (Wang et al, 1999; Ding et al, 2007).

3. Experiments and Results

3.1 Test configuration

The data was collected using two dual frequency Leica 530 GPS receivers with 1Hz sampling rate and one BEI C-MIGITSII (DQI-NP) INS unit. One of the receivers was set static and the other one along with DQI-NP were set on the top of a ground vehicle. The generated moving trajectory of the vehicle from ambiguity fixed solution has been shown in Figure 2.

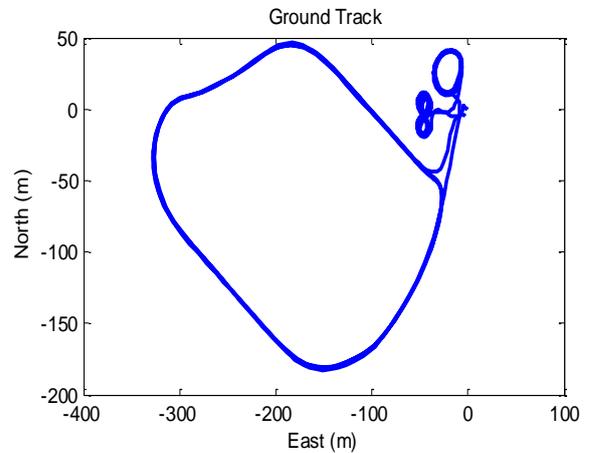


Figure 1: Ground track

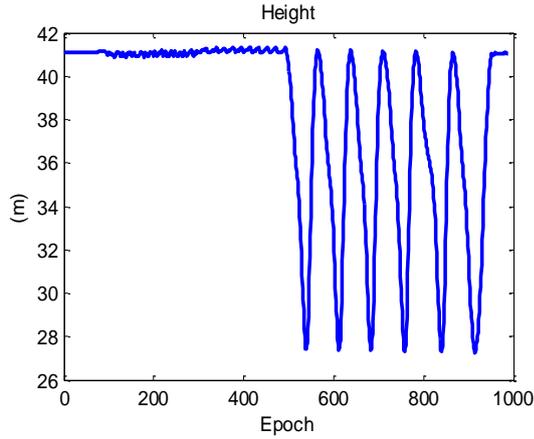


Figure.2: Height profile

To process the data, an integrated GPS/INS system has been implemented for data fusion and error estimation based on complementary form of the extended Kalman filter, and INS psi-angle error model. The dynamic model includes twenty four states, i.e. nine navigation solution errors of three dimensional position, velocity and attitude, six accelerometer error modelling parameters (bias and scale factors for each axis), three gyro drifts, three gravity uncertainty errors, and three lever arm errors.

$$\begin{aligned} \delta \dot{\mathbf{v}} &= -(\boldsymbol{\omega}_{ie} + \boldsymbol{\omega}_{in}) \times \delta \mathbf{v} - \delta \boldsymbol{\psi} \times \mathbf{f} + \delta \mathbf{g} + \nabla \\ \delta \dot{\mathbf{r}} &= -\boldsymbol{\omega}_{en} \times \delta \mathbf{r} + \delta \mathbf{v} \\ \delta \dot{\boldsymbol{\psi}} &= -\boldsymbol{\omega}_{in} \times \delta \boldsymbol{\psi} + \boldsymbol{\varepsilon} \end{aligned} \quad (22)$$

$\delta \mathbf{v}$, $\delta \mathbf{r}$ and $\delta \boldsymbol{\psi}$ are the velocity, position, and attitude error vectors respectively;

$\boldsymbol{\omega}_{ie}$ is the Earth rate vector;

$\boldsymbol{\omega}_{in}$ is the angular rate vector of the true coordinate system with respect to the inertial frame;

$\boldsymbol{\omega}_{en}$ is the angular rate vector of the true coordinate system with respect to the Earth;

∇ is the accelerometer error vector;

$\delta \mathbf{g}$ is the error in the computed gravity vector;

$\boldsymbol{\varepsilon}$ is the gyro drift vector;

\mathbf{f} is the specific force vector.

Table 1: DQI-NP's technical data

	Gyro	Accelerometer
Bias	5 deg/hr	500 μg
Scale factor	500 ppm	800 ppm
Random walk/ white noise	0.035 deg/sqrt(hr)	180 μg /sqrt(hr)

Table 1 shows the DQI-NP's technical data for reference. The specified parameters were used in setting up the default Q values in the filtering process. The noise of GPS derived positions was considered to be non-

correlated between axes with magnitude at the centimetre level. The diagonal elements of default R was set to 3cm*3cm. A default window size of 15 is used in calculating innovation and residual covariance matrix.

3.2 Covariance analysis (RMS) of the estimated position, velocity and attitude

First, the filtering performance was evaluated using covariance analysis. The covariance matrix of state estimation indicates the theoretical performance of the Kalman filter. It is often used for checking the stability, and predicting filtering performance during the design phase. When a state covariance is growing without bound, the filter is said to be diverging. A well performed filter would have its state covariance settled to a stable value after the initial transient period when there are no more external disturbances. For a linear conventional Kalman filter, its state covariance matrix is totally determined from given initial conditions, and P and Q values without direct link to the actual measurements. In contrast, the covariance matrix of an adaptive Kalman filter may change during the filtering process because it is partially derived from actual measurements which are different in different tests.

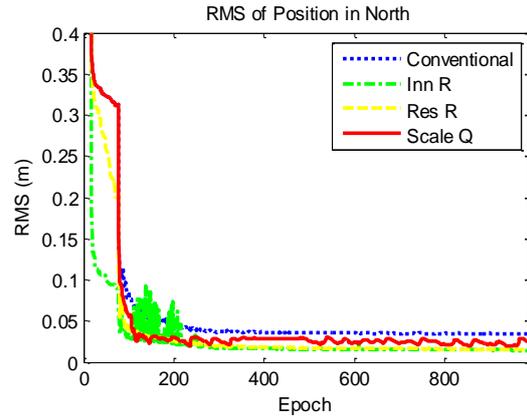


Figure.3: Position RMS in North direction

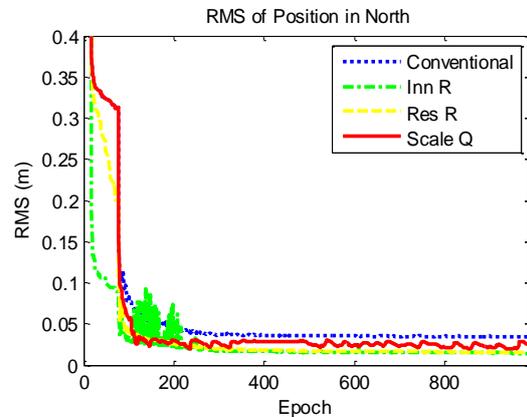


Figure.4: Position RMS in East direction

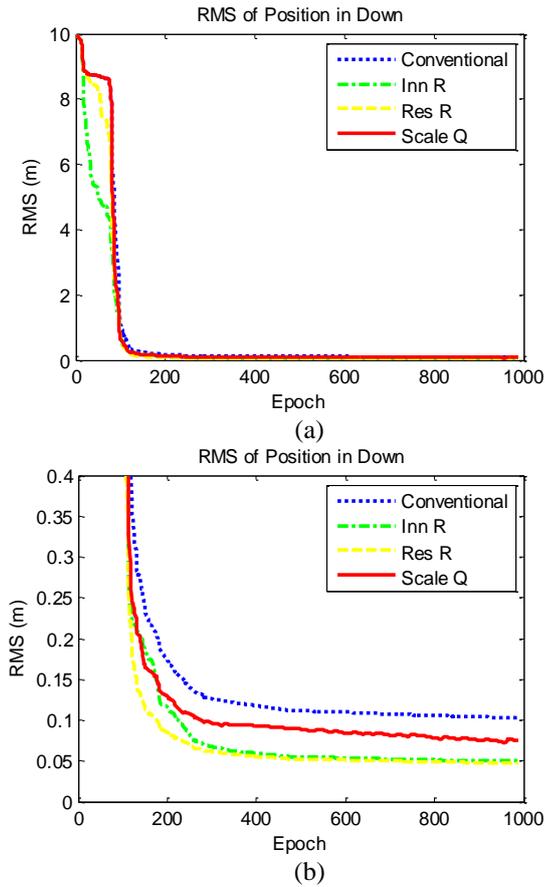


Figure.5: Position RMS in Down direction (a) in full vertical scale; (b) in enlarged vertical scale.

Figures 3, 4 and 5 show the RMS of the covariance of the position states derived from the diagonal elements of the estimated state covariance matrix $\hat{\mathbf{P}}_k$. The covariance analysis of velocity and attitude are not shown here since they have the similar trends to the position states. In general, one can note that all the position components (XYZ) are converged below 0.1m quickly. In comparison between conventional and the adaptive cases, the adaptive ones are slightly better than those obtained from the conventional ones. It is obvious that the innovation based adaptive method is better in terms of converging speed. However, some spikes appear between 100 and 200 seconds, and 600 and 700 seconds with innovation based method, indicating a problem in smooth estimation. This issue will be discussed later in detail. The residual based method also outperforms conventional and Q scale methods in converging speed. Overall, all adaptive methods demonstrate stable estimation characteristics, and their stable RMS values are slightly better than the conventional one.

3.3 Innovation and residual analysis

In order to evaluate the actual filtering performance, the innovation series generated during KF filtering are

examined. In general, the innovation and residual analysis provides a more objective view of the actual filtering performance. If the dynamic system was perfectly modelled in the Kalman filter, both the innovation and residual series should be zero-mean white noise processes. Unlike the state covariance matrix generated by the Kalman filter self, the statistics of innovation and residual series are independent and faithful filtering quality indicators.

The mean and standard deviation (STD) values of conventional and adaptive KFs were listed in Table 2 and 3. It can be seen that innovation and residual based R estimation methods generated better accuracy than the conventional one and Q scaling method, and residual based method is slightly better than the innovation based method. Q scaling method performs worse than the conventional one. The most likely reason is that the default settings as given in this test are already very near to the best values according to our knowledge. Plus the tuning capacity of Q scaling is relying on only one parameter, which is to improve the robustness in adaptation. It may be as sensitive in the tuning process as the other two adaptive methods.

Table 2: Comparing the mean values of the innovation series generated from different filtering methods

	Mean Values		
	X (m)	Y (m)	Z (m)
Conventional	-0.0020	0.0009	-0.0026
Inn R	-0.0011	0.0005	-0.0015
Res R	-0.0013	0.0007	-0.0015
Scale Q	-0.0097	0.0029	-0.0053

Table 3: Comparing the STD values of the innovation series generated from different filtering methods

	STD Values		
	X (m)	Y (m)	Z (m)
Conventional	0.0495	0.0588	0.0169
Inn R	0.0454	0.0490	0.0201
Res R	0.0388	0.0415	0.0192
Scale Q	0.0722	0.0765	0.0391

3.4 Influence of window sizes

Another comparison was conducted using four different window sizes (15, 30, 60 and 120 seconds). This is to test the sensitivity of estimated parameters to different window sizes used. Earlier studies such as (Mohamed and Schwarz, 1999; Ding et al, 2007) found that the small window size could lead the filter to diverge whereas the larger the moving window, the less biases are the estimates. However, very large window size may cause the adaptive filter losing the ability of adaptation. So choosing an optimal window size should consider the application circumstances, and some defined criteria (Wang et al, 1999). In this case, the number of unknown

estimates and the measurements should also be considered.

3.4.1 R estimation

Tables 4, 5, 6, and 7 show the statistics of the estimation using the innovation and residual based R estimation with windows sizes of 15, 30, 60 and 120. In general, no significant difference with innovation mean and STD values is observed when window size is changed from 15 to 120. Slightly, window size of 120 seconds performs best for innovation based R estimation; and window size of 60 seconds performs best for residual based R estimation.

Table 4: Comparing the mean values of the innovation series generated from the innovation based R estimation

Window sizes	Mean Values		
	X (m)	Y (m)	Z (m)
15	-0.0011	0.0005	-0.0015
30	-0.0011	0.0008	-0.0015
60	-0.0011	0.0005	-0.0015
120	-0.0015	0.0006	-0.0015

Table 5: Comparing the STD values of the innovation series generated from the innovation based R estimation

Window sizes	STD Values		
	X (m)	Y (m)	Z (m)
15	0.0454	0.0490	0.0201
30	0.0462	0.0461	0.0201
60	0.0419	0.0424	0.0198
120	0.0363	0.0397	0.0186

Table 6: Comparing the mean values of the innovation series generated from the residual based R estimation

Window sizes	Mean Values		
	X (m)	Y (m)	Z(m)
15	-0.0013	0.0007	-0.0015
30	-0.0013	0.0007	-0.0015
60	-0.0013	0.0006	-0.0015
120	-0.0014	0.0007	-0.0015

Table 7: Comparing the mean values of the innovation series generated from the residual based R estimation

Window sizes	STD Values		
	X (m)	Y (m)	Z (m)
15	0.0388	0.0415	0.0192
30	0.0385	0.0414	0.0192
60	0.0375	0.0407	0.0188
120	0.0380	0.0415	0.0183

3.4.2 Q estimation

Figure 6 shows the Q scaling factors calculated with different window sizes. Because of differences in window size, the starting points of the displayed values are different. After the adaptive algorithm becomes active, a transient period is accompanied by an overshoot of the estimations. Then gradually the estimated values converge to one. This indicates the adaptation is stable. When the scaling factor equals one, the Q actually has no more change and the filter behaves like a conventional filter. From the figure it can be seen that large window size generates a smoother estimation. The large overshoot at the beginning of estimation could be caused by initial estimation errors, which can be reduced using more sophisticated control rules.

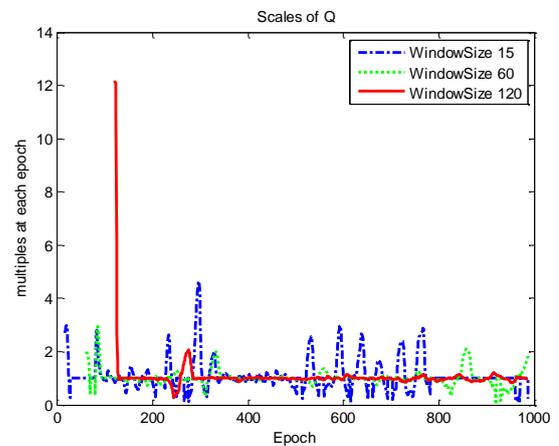


Figure. 6: Histogram of the estimated scaling factor

Table 8: Comparing the mean values of the innovation series generated from the Q scaling method

Window size	Mean Values		
	X (m)	Y (m)	Z (m)
15	-0.0097	0.0029	-0.0053
30	-0.0017	0.0033	-0.0003
60	-0.0045	-0.0036	0.0090
120	0.0005	0.0044	-0.0048

Table 9: Comparing the STD values of the innovation series generated from Q scaling method

Window size	STD		
	X (m)	Y (m)	Z (m)
15	0.0722	0.0765	0.0391
30	0.0651	0.0707	0.0557
60	0.0640	0.0755	0.1057
120	0.0801	0.0780	0.0273

Tables 8 and 9 show the statistics of the estimation using Q scaling method with window sizes of 15, 30, 60 and 120. By comparing the STD values, window size of 60

seconds generates best results, even though window size 120 generates the smoothest scale factor estimation.

3.5 Simultaneously adaptation of Q and R

Full estimation of Q and R based on covariance matching is questionable. This is because, as indicated by Equations (14) and (17), these two values are relying on each other in the covariance matching equation. Several tests were carried out on combining Q scaling and R estimation. Although stable results have been obtained, its performance is not better than those obtained from using individual techniques.

3.6 Simultaneously adaptation of Q and R

- a) As mentioned above, one drawback of using innovation based R estimation is that the positive definite nature of R cannot be guaranteed when equation (14) is used, especially when the innovation covariance is estimated within a relatively small sampling window. This trouble can easily cause the filter diverged when handling real field data. To deal with this problem, a compromised solution is adopted in this work. Each time when a new R matrix is calculated, its elements are checked and forced to be larger than 1mm^2 before it is used for filtering process. This ad hoc treatment could be the reason that caused the spikes mentioned in Section 3.2. The residual based R estimation has overcome this drawback.
- b) All the three adaptive methods discussed here are derived from the principle of covariance matching. In the process, when R is to be estimated, Q is always supposed to be perfectly known, and vice versa. Since perfect Q or R is not known in real data processing, they can only be set based on the best knowledge. In the implementation, both innovation and residual based R estimates exhibit high sensitivity to the errors of Q setting. Proper operation of the Q scaling method is also relying on reasonable R setting, even though it is bit more robust than the other two adaptive methods.

4. Concluding Remarks

Evaluation of adaptive Kalman filters for GPS/INS integration is essential for selecting the proper method for adapting measurement noise matrix R and dynamic noise covariance matrix Q . Over the past decades, constant matrices are commonly used in the conventional Kalman filter. Various approaches have already been implemented for different GPS/INS integration algorithms. However, choosing the best method that provides reliable results is still a controversial issue.

In this paper, three approaches based on covariance matching principles of adaptive Kalman filter were implemented in loose GPS/INS integration system for adapting R and Q . All these methods are based on calculations of the covariance matrix of the innovation and residual series with a moving sampling window. The results show the improvements of adaptive Kalman filter methods in different scenarios over the conventional Kalman filter, and the comparison of the performance of different filtering schemes. Overall, the residual based R estimation is considered to be outperforming the innovation based R estimation. The Q scaling method is considered more robust to modelling deviations, while its filtering accuracy is not as good as the performance of using the residual based R estimation in certain cases. Changing of the moving window size has only minor influence on the adaptive filtering performance when the application dynamics is not high. Some implementation issues have been discussed, which need further investigation.

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Biography

Ali Almagbile is currently a PhD candidate in the school of Surveying and Spatial Information System at the University of New South Wales. His research interest is in multi-sensor integration systems for unmanned aerial vehicle (UAV) navigation and mapping. He obtained a Master of geography at the University of Jordan in 2003.