Journal of Global Positioning Systems (2012)
Vol.11, No.1 : 46-57
DOI: 10.5081/jgps.11.1.46

Journal of Global Positioning Systems
2012

New Outlier Separability Test and Its Application in GNSS Positioning

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Abstract

With the increasing automation of measurement, adjustment, outlier detection and the consequential use of the results for real-time applications, reliable methods to detect and mitigate an outlier are required. However, it is frequently advised that the outlier test should not be used as a means to automatically reject an outlier. One of the reasons for this is that the outlier detection test at times can identify a wrong measurement. To address this issue, this paper proposes a new outlier separability test to confirm that the identified outlier, by the outlier test, can be confidently rejected as the outlier. In addition, the Minimally Separable Bias and the separability multiplying factor are also obtained for the proposed outlier separability test. With the initial comparisons between the proposed method with the reapplication of the global model test method and multiple hypothesis method, the advantages of the new outlier separability test are demonstrated.

Keywords: Outliers; Statistical Testing; Separability, Minimal Separable Bias (MSB); GNSS

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1. Introduction

An outlier detection test is often used to identify an outlying measurement. However, despite this it is often advised, with the use of the outlier test, to simply flag suspected outlying measurements for further investigations, either in the regular Gauss-Models (Baarda, 1968; Pope, 1976) or in the singular ones (Wang and Chen, 1999). One of the reasons for this is that the outlier test at times can incorrectly identify a wrong measurement as the outlying measurement (Stefanovic, 1980; Hawkins, 1980; Krarup et al., 1980; Kok, 1984). Hawkins (1980) refers to such an event as a Type III error where the null hypothesis is correctly rejected, due to the presence of an outlier; however, the wrong measurement is rejected as the outlier. Therefore, in the literature, such occurrence is also called as swapping, which may have an impact in navigation system performance (e.g., Wang and Ober, 2009).

Due to the increasing need to carry out outlier detection and rejection automatically, several strategies have been proposed to ensure that the outlying measurement can be detected and confidently separated from the rest of the measurements. The existing strategies include the reapplication of the global model test (Stefanovic, 1980; Parkinson & Axelrad, 1988; Lee et al., 1996; Lee and Van Dyke, 2002), the multiple hypothesis method (Förstner, 1983; Li, 1986), the differencing outlier statistics method (Lee, 1995; Kelly, 1998) and the Bayesian approach (Pervan et al., 1996; 1998).

In the reapplication of the global model test method, each measurement is removed iteratively and the global model test is reapplied. If only one of the resulting global model tests passes, it is concluded that the corresponding measurement can be confidently separated (Stefanovic, 1980; Parkinson and Axelrad, 1988; Lee et al., 1996; Lee and Van Dyke, 2002). However, the reapplication of the global model test method does not involve the explicit calculation of the confidence level at which the measurement has been separated from the remaining measurements.

The multiple hypothesis method proposed by Förstner (1983) and Li (1986) is based on the testing of the null hypothesis against two alternative hypotheses with the bivariate normal distribution formed from two single outlier detection statistics. In addition the method lso enables the calculation of the Minimal Separable Bias (MSB) for a particular separability confidence level between a pair of outlier detection statistics. With this principle, several studies have been carried out to analyse the outlier separability with various satellite constellations (Hewitson and Wang, 2006; 2007).

The differencing outlier statistics method is based on the difference between a pair of outlier detection statistics following a normal distribution. Then, if the magnitude of the bias is known, the confidence level at which the two outlier statistics can be separated is obtained (Lee, 1995; Kelly, 1998). However, in practice the magnitudes of the biases are unknown. Nevertheless, based on the assumption that the bias is equal to the Minimal Detectable Bias (MDB), Kelly (1998) adopted the method to exclude all the measurements that cannot be separated with a given confidence level.

Pervan et al. (1996; 1998) obtained the probability of identifying a measurement corresponding with the outlier using the Bayesian theory. However, the method is dependent on the initial estimates of the probabilities of measurements being contaminated by outliers and assumes that the outliers conform to a normal distribution.

Hence, to overcome the limitations from the existing methods discussed above, a new outlier separability test is highly desirable. In this paper, a practical strategy of separating an outlying measurement at a set confidence level is developed. The new separability test and the associated measures of separability are then applied to GNSS positioning examples to demonstrate the advantages of the proposed outlier detection and identification strategy.

2. The Outlier Detection Test

Assuming that there is no outlier in measurements, the least squares estimation of the unknown parameters from the measurements can be based on the following functional and stochastic models:

$$v = Ax - l \tag{1}$$

$$\Sigma = {\sigma_0}^2 Q = {\sigma_0}^2 P^{-1} \tag{2}$$

where v is the vector of residuals; A is the n by t design matrix; x the vector of t parameters and ℓ is the n by 1 measurement vector. Σ is the positive definite variance covariance matrix of the measurements. ${\sigma_0}^2$ is the a priori variance factor, Q is the cofactor matrix, and P is the weight matrix.

In order to test if there is an outlier in the i^{th} measurement, the hypotheses to be tested are

$$\begin{aligned} \mathrm{H_o}&: \mathrm{E}\{\ell\} = Ax \\ \mathrm{H_a}&: \mathrm{E}\{\ell\} = Ax + h_i \nabla s_i \end{aligned} \tag{3}$$

where ∇s_i is the assumed outlier in the i^{th} measurement and h_i is a vector of zeros with the i^{th} element being equal to one, The outlier detection test commonly employed to identify an outlier can be derived from the mean shift model

$$v_i = [A \quad h_i] \begin{bmatrix} x \\ \nabla s_i \end{bmatrix} - \ell \tag{4}$$

Solving Equations (4) and (2) for the outlier using the partitioned matrixes yields (e.g., Wang and Chen, 1994a; 1999; Kok, 1984; Knight et al., 2010)

$$\widehat{\nabla s}_i = ({h_i}^{\mathrm{T}} P Q_v P h_i)^{-1} {h_i}^{\mathrm{T}} P Q_v P \ell \tag{5}$$

where $Q_v = P^{-1} - A(A^T P A)^{-1} A^T$ is the cofactor matrix of the estimated residuals from the original Gauss-Markov model without the mean shift, presented by Equations (1) and (2).

In addition, the variance of the estimated outlier in the i^{th} measurement can be evaluated via the error propagation as follows

$${\sigma_{\widehat{\nabla s}_i}}^2 = {\sigma_0}^2 ({h_i}^{\mathrm{T}} P Q_v P h_i)^{-1} \tag{6}$$

Therefore, the outlier detection test statistic for the i^{th} measurement can be formed as (Baarda 1968; Kok 1984)

$$w_i = \frac{\widehat{\nabla s}_i}{\sigma_{\widehat{\nabla s}_i}} = \frac{{h_i}^{\mathrm{T}} P Q_v P \ell}{\sigma_0 \sqrt{{h_i}^{\mathrm{T}} P Q_v P h_i}} \sim \mathrm{N}(0,1) \tag{7}$$

Likewise, in order to test if there is an outlier in the k^{th} measurement, the hypotheses to be tested are

$$\begin{aligned} \mathrm{H_o}&: \mathrm{E}\{\ell\} = Ax \\ \mathrm{H_a}&: \mathrm{E}\{\ell\} = Ax + h_k \nabla s_k \end{aligned} \tag{8}$$

Similarly, the outlier statistic can also be formed for the k^{th} measurement using the linear model

$$v_k = [A \quad h_k] \begin{bmatrix} x \\ \nabla s_k \end{bmatrix} - \ell \tag{9}$$

where h_k is the vector of zeros with the k^{th} element being equal to one, and ∇s_k is the outlier in the k^{th} measurement. Therefore, the k^{th} outlier statistic can be formed as

$$w_k = \frac{\widehat{\nabla s}_k}{\sigma_{\widehat{\nabla s}_k}} = \frac{{h_k}^{\mathrm{T}} P Q_v P \ell}{\sigma_0 \sqrt{{h_k}^{\mathrm{T}} P Q_v P h_k}} \sim \mathrm{N}(0,1) \tag{10}$$

In addition, the correlation coefficient between the i^{th} and k^{th} outlier detection statistics can be obtained as (Förstner 1983)

$$\rho_{ik} = \rho_{ki} = \frac{{h_k}^{\mathrm{T}} P Q_v P h_i}{\sqrt{{h_k}^{\mathrm{T}} P Q_v P h_k} \sqrt{{h_i}^{\mathrm{T}} P Q_v P h_i}} \tag{11}$$

3. The Minimally Detectable Bias

Unfortunately, the outlier statistic test does not always detect the presence of a bias with a given confidence level. However, as the size of the bias increases the probability of the bias being detected by the outlier detection statistic also increases. Hence, to gain an appreciation for how large a bias must become before it can be detected, the Minimally Detectable Biases (MDBs) are formulated for a given set of Type I and Type II errors (Teunissen 1990; 1991). As shown in Figure 1, for the given Type I and Type II errors, α_d and

β_d respectively, if the tiny contribution of the integral between -∞ and $N_{\alpha_d/2}(0,1)$ under Ha is ignored, the corresponding mean shift in the outlier statistic can be obtained as follows

$$\delta_d = N_{1-\alpha_d/2}(0,1) - N_{\beta_d}(0,1) \quad (12)$$

In addition, the expected mean shift of the outlier statistic when there is an outlier in the i^{th} measurement can be obtained as

$$E\{w_i\} = \frac{\nabla s_i \sqrt{h_i{}^T P Q_v P h_i}}{\sigma_0} \quad (13)$$

Therefore, if the expected shift of the outlier statistic is set as δ_d, the MDB of the i^{th} measurement can be obtained as

$$MDB_i = \frac{\delta_d \sigma_0}{\sqrt{h_i{}^T P Q_v P h_i}} \quad (14)$$

Based on the MBS above, other simplified reliability measures can also be derived (e.g., Wang and Chen, 1994a; 1994b; 1999)

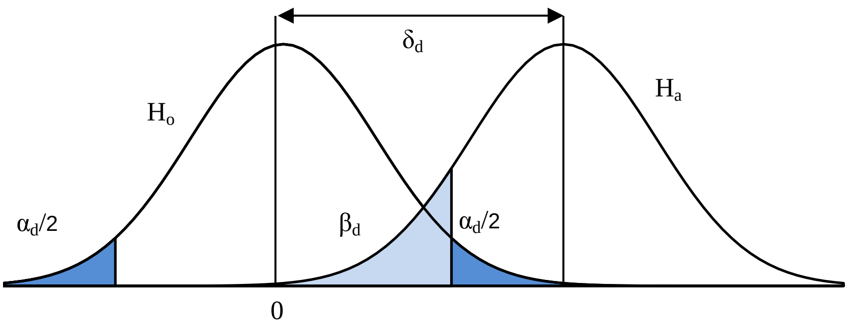

Figure 1: The Outlier Testing and the Null and Alternate Hypotheses

4. The Outlier Separability Test

In the outlier detection procedure, the largest outlier detection statistic is to be identified and then, the measurement associated with this largest detection statistic is the most likely outlying measurement. Therefore, the larger the difference between any two outlier detection statistics, the smaller the probability of identifying a wrong measurement.

However, the difference between the two outlier detection statistics is dependent on the correlation coefficient between the two outlier detection statistics. Statistically, both outlier detection statistics are expected to be positive or negative if the two statistics are positively correlated, and likewise one positive and the other one negative if the two statistics are negatively correlated. Therefore, a separability test statistic can be formed as

$$w_{ik} = \begin{cases} w_i - w_k & \rho_{ik} \geq 0 \\ w_i + w_k & \rho_{ik} < 0 \end{cases} \quad (15)$$

However, since the addition and subtraction of two normal distributions results in a normal distribution, w_{ik} also has a normal distribution. Hence, to obtain the variance of the separability statistic the error propagation law can be applied to Equation (15) as follows

$$\sigma_{w_{ik}}{}^2 = 2 - 2\rho_{ik} \quad \rho_{ik} \geq 0 \quad (16)$$

for the subtraction of the i^{th} and k^{th} outlier detection statistics, and

$$\sigma_{w_{ik}}{}^2 = 2 + 2\rho_{ik} \quad \rho_{ik} < 0 \quad (17)$$

for the addition of the i^{th} and k^{th} outlier detection statistics.

Therefore, the separability test statistic can be converted to the new standardised statistic as:

$$J_{ik} = \begin{cases} \dfrac{w_i - w_k}{\sqrt{2 - 2\rho_{ik}}} & \rho_{ik} \geq 0 \\ \dfrac{w_i + w_k}{\sqrt{2 + 2\rho_{ik}}} & \rho_{ik} < 0 \end{cases} \quad (18)$$

where the critical value is given by $N_{1-\alpha_s/2}(0,1)$, based on α_s being the Type I error in the separability test with the i^{th} and k^{th} outlier statistics. If $|J|$ is smaller than the critical value then the null hypothesis

$$H_o : E\{J\} = 0 \quad (19)$$

is accepted. Therefore, it is concluded that with the given confidence level (1-α_s), there is not sufficient evidence to separate two outlier detection statistics. If $|J|$ is larger than the critical value, the alternate hypothesis

$$H_a : E\{J\} \neq 0 \quad (20)$$

is accepted, and thus, the two outlier statistics can be separated from each other with a given confidence level of 1-α_s.

The new separability test discussed above is named as the **JN Test.**

5. The Minimally Separable Bias Based on the JN-Test

Similar to the outlier detection test where the corresponding MDB can be obtained, the Minimally Separable Bias (MSB) can also be obtained for the above separability test (i.e., **JN-Test**), for a given set of Type I and Type II errors. It is expected that, as the size of the bias increases, the probability of the outlier being incorrectly identified decreases. Hence, as shown in Figure 2, for the given Type I and Type II errors, α_s and β_s respectively, if the tiny contribution of the integral between -∞ and $N_{\alpha_s/2}(0,1)$ under Ha is ignored, the expected mean shift in the separability test can be obtained as follows

$$\delta_s = N_{1-\alpha_s/2}(0,1) - N_{\beta_s}(0,1) \qquad (21)$$

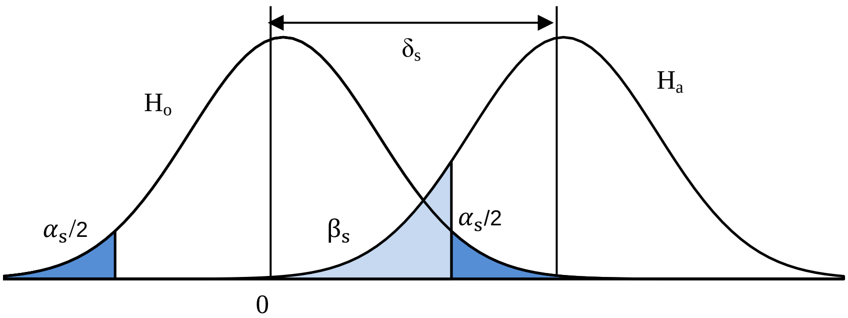

Figure 2: The Separability Test and the Null and Alternate Hypothesis

The expected mean shift in the separability test statistic can also be obtained based on an outlier in the i^{th} measurement as

$$E\{J_{ik}\} = \begin{cases} \frac{\nabla s_i}{\sigma_0\sqrt{2}}\sqrt{h_i{}^T P Q_v P h_i (1-\rho_{ik})} & \rho_{ik} \geq 0 \\ \frac{\nabla s_i}{\sigma_0\sqrt{2}}\sqrt{h_i{}^T P Q_v P h_i (1+\rho_{ik})} & \rho_{ik} < 0 \end{cases} \qquad (22)$$

which can be simplified to

$$E\{J_{ik}\} = \frac{\nabla s_i}{\sigma_0\sqrt{2}}\sqrt{h_i{}^T P Q_v P h_i (1-|\rho_{ik}|)} \qquad (23)$$

Hence, it can be seen that when the size of the outlier is zero, or $|\rho_{ik}|$ is equal to one, the separability statistics expectation is also zero, which is the situation in which the outlier cannot be confidently separated. However, as the size of the outlier increases, the separability statistic becomes increasingly noncentral. Hence, with the expected mean shift of the separability statistic δ_s, as described by Equation (21), the MSB for the i^{th} and k^{th} measurements can be obtained as

$$MSB_{ik} = \frac{\delta_s \sigma_0 \sqrt{2}}{\sqrt{h_i{}^T P Q_v P h_i (1-|\rho_{ik}|)}} \qquad (24)$$

Likewise, the MSB for the k^{th} and i^{th} observations when the bias is considered in the k^{th} measurement can be obtained as

$$MSB_{ki} = \frac{\delta_s \sigma_0 \sqrt{2}}{\sqrt{h_k{}^T P Q_v P h_k (1-|\rho_{ki}|)}} \qquad (25)$$

6. The Separability Multiplying Factor for the JN-Test

When using the outlier detection test, together with an outlier separability test, to identify and confidently separate an outlier, the outlier detection and identification process is related to, not only the MDBs but also the MSBs. Hence, to obtain a more complete measure of the outlier identification and separation performances, the separability multiplying factor, $k_{\rho_{ik}}$, can be defined as,

$$MSB_{ik} = k_{\rho_{ik}} MDB_i \qquad (26)$$

If the MDB and MSB, shown by Equations (14) and (24) respectively, are substituted into Equation (26), a unique expression for the separability multiplying factor can be obtained as follows

$$k_{\rho_{ik}} = \frac{\delta_s\sqrt{2}}{\delta_d\sqrt{1-|\rho_{ik}|}} \qquad (27)$$

Hence, it can be seen that the outlier separability multiplying factor is dependent on the probabilities of Types I and II errors selected, for the outlier detection (α_d, β_d) and for the separability test (α_s, β_s), as well as the correlation coefficient between the two outlier detection statistics used. A graphical representation of this relationship is displayed in Figure.3 for various correlation coefficients and δ_s/δ_d ratios.

However, since the probabilities of Types I and II errors are the predefined values, then the separability multiplying factor is primarily dependent on the correlation coefficient. Therefore, small correlation coefficients between the outlier statistics are desired in order to achieve the smallest separability multiplying factors.

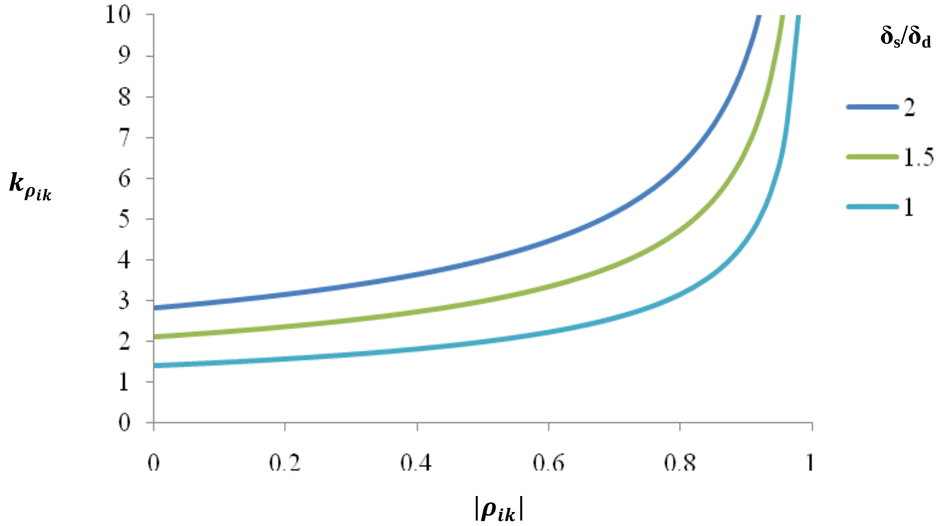

Figure 3: Separability Multiplying Factors

7. Numerical Examples

As an example, we consider the single point positioning situation displayed in Figure 4. The data used here was collected from Minot, North Dakota, USA on the 18th August 2008.

Based on α_d and β_d being 0.1% and 20% respectively, the MDBs can be obtained from Equation (14) and are displayed in Table 1. It can be seen that the largest MDB is 65m for SV26 and the smallest MDB is 21m for SV28.

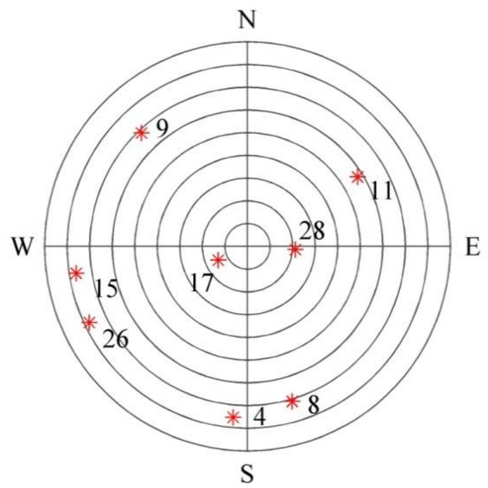

Figure 4: Sky Plot

Table 1: Minimal Detectable Bias

SV	4	8	9	11	15	17	26	28
MDB (m)	61.01	55.64	56.36	43.89	62.33	26.38	65.23	20.90

7.1 An outlier of 500m in SV17

If an outlier of 500m is now introduced into SV17, which is larger than the MDB of 26m, then the outlier test consequently fails. However if the outlier detection statistics are examined, as shown in Table 2, it is found that the largest outlier statistic corresponds with SV28 instead of SV17. Hence, if no separability test were employed, a Type III error would be committed here, with the wrong measurement being rejected.

If a separability test is employed, the separability statistics can be computed based on the outlier statistics in Table 2 and their correlation coefficients shown in Table 3. In addition if the accepted probability of committing a Type I error, α_s, is set as 0.1%, the critical value of the separability test can be obtained as 3.291.

Therefore, from the separability statistics in Table 4, it can be seen that the pairs of satellites that cannot be confidently separated include SV15 and SV8, SV26 and SV8, and SV28 and SV17. However, since it is expected that the largest outlier detection statistic (in absolute value) will correspond with the outlier (Stefanovic 1980), it is simply required to ensure that SV28 can be confidently separated from the other outlier detection statistics, particularly the second largest one (in absolute value). Therefore, since SV28 cannot be confidently separated from SV17 it is concluded that there is too much doubt as to which measurement contains the outlier at the significance level of 0.1%.

Table 2: Outlier Statistics for an Outlier of 500m in SV17

SV	4	8	9	11	15	17	26	28
w	1.099	15.355	-30.094	69.313	-19.524	79.456	-15.500	-79.458

Table 3: Correlation Matrix

ρ_{ik}		SV_k							
		4	8	9	11	15	17	26	28
SV_i	4	1	-0.7278	0.1389	0.0283	-0.1888	0.0217	-0.2466	-0.0091
	8		1	0.3961	-0.1606	-0.1751	0.1890	-0.2777	-0.1989
	9			1	-0.7696	-0.5079	-0.3774	-0.3406	0.3778
	11				1	0.0327	0.8717	0.0900	-0.8709
	15					1	-0.2416	-0.2569	0.2418
	17						1	-0.2038	-0.9999
	26							1	0.1996
	28								1

Table 4: Separability Statistics with the JN-Test for an Outlier of 500m in SV17

J_{ik}		SV_k							
		4	8	9	11	15	17	26	28
SV_i	4		22.302	23.769	-48.932	-14.465	-56.019	-11.732	-55.661
	8	22.302		41.354	65.345	-3.245	-50.331	-0.120	-50.643
	9	-23.769	-41.354		57.773	-50.013	44.236	-39.703	44.252
	11	48.932	65.345	57.773		63.871	-20.023	62.868	-19.965
	15	-14.465	-3.245	-50.013	-63.871		48.662	-28.730	48.669
	17	56.019	50.331	44.236	20.023	48.662		50.683	-0.201
	26	-11.732	-0.120	-39.703	-62.868	-28.730	50.683		50.551
	28	-55.661	-50.643	-44.252	-19.965	-48.669	-0.201	-50.551	

The separability of the outlier detection statistics can also be empirically evaluated using the reapplication of the global model test method (Stefanovic, 1980; Parkinson and Axelrad, 1988; Lee et al, 1996; Lee and Van Dyke, 2002). If the global model test statistic is computed, it is found that the test fails with a value of 6313.981, which is significantly larger than the critical value of 9.488 at the significance level of 5%. If each measurement is now removed and the global model test statistic is recomputed, the resulting values are displayed in Table 5. In addition, at the 5% significance level the critical value is also reduced to 7.815.

Hence, from Table 5 it can be seen that, in addition to SV28 passing, SV17 also passes. Therefore, it is concluded that the outlier cannot be confidently identified.

Table 5: Global Model Statistics for an Outlier of 500m in SV17

SV	$v^{\mathrm{T}}Pv/\sigma_0{}^2$
4	6312.774
8	6078.197
9	5408.240
11	1509.351
15	5932.778
17	0.792
26	6073.734
28	0.379

Table 6: Probabilities of Type I Errors

α_s(%)		SV_k							
		4	8	9	11	15	17	26	28
SV_i	4		6.37	0.34	0.20	0.42	0.19	0.56	0.18
	8	6.37		1.16	0.37	0.40	0.43	0.65	0.45
	9	0.34	1.16		8.04	2.02	1.06	0.88	1.06
	11	0.20	0.37	8.04		0.20	14.77	0.27	14.69
	15	0.42	0.40	2.02	0.20		0.55	0.59	0.55
	17	0.19	0.43	1.06	14.77	0.55		0.46	48.93
	26	0.56	0.65	0.88	0.27	0.59	0.46		0.45
	28	0.18	0.45	1.06	14.69	0.55	48.93	0.45	

In addition, the separability of the outlier statistics can also be evaluated using the differencing outlier detection statistics method (Lee, 1995; Kelly, 1998). In this method, the probability of committing a Type I error between each pair of measurements is computed based on the MDBs as shown in Table 1, and the resulting values are tabulated in Table 6. From Table 6 it can be seen that all the probabilities of committing a Type I error are greater than the accepted probability of 0.1%. This is particularly so for SV28 and SV17 with a probability of committing a Type I error at 49%. Hence, the outlier of the size just as large as the associated MDB cannot be confidently separated.

7.2 An outlier of 1000m in SV17

If the outlier added to SV17 is increased to 1000m instead of 500m, it is found that, as shown in Table 7, the identified outlier now corresponds with the true outlier in SV17.

Table 7: Outlier Statistics for an Outlier of 1000m in SV17

SV	4	8	9	11	15	17	26	28
w	2.797	30.159	-59.675	137.607	-38.449	157.797	-31.465	-157.793

If the separability of the outlier detection statistics is evaluated, as shown in Table 8, it is found that now only the pairs if the outlier statistics of SV26 and SV8, and SV28 and SV17, are considered to be inseparable. However since SV17 is the identified outlier statistic, it is required that the second largest outlier detection statistics (in absolute value) be separable from SV17. Since this is not the case, it is still concluded that there is too much doubt as to which measurement contains the outlier.

Table 8: Separability Statistics for an Outlier of 1000m in SV17

J_{ik}		SV_k							
		4	8	9	11	15	17	26	28
SV_i	4		44.673	47.602	-96.703	-27.990	-110.811	-23.355	-110.096
	8	44.673		81.737	129.477	-6.454	-100.219	-1.087	-100.834
	9	-47.602	-81.737		114.807	-98.904	87.940	-79.366	87.962
	11	96.703	129.477	114.807		126.579	-39.861	125.326	-39.729
	15	-27.990	-6.454	-98.904	-126.579		96.905	-57.348	96.912
	17	110.811	100.219	87.940	39.861	96.905		100.112	0.307
	26	-23.355	-1.087	-79.366	-125.326	-57.348	100.112		99.846
	28	-110.096	-100.834	-87.962	-39.729	-96.912	0.307	-99.846	

Table 9: Global Model Statistics for an Outlier of 1000m in SV17

SV	$v^{T}Pv/{\sigma_0}^2$
4	24892.847
8	23991.175
9	21338.716
11	5962.501
15	23422.194
17	0.792
26	23910.523
28	2.049

If the separability using the reapplication of the global model test is considered, the statistics obtained are shown in Table 9. Since two global model statistics pass, it can hence also be concluded that there is too much doubt as to which measurement contains the outlier.

In addition, if the separability using the differencing outlier statistics method is considered, it is found that the probabilities of committing a Type I error are the same as in Table 6. This is due to the fact that the differencing outlier statistics method is independent of the sampled outlier detection statistics. Hence, due to the limitation of this method, the conclusion will always be made that any outlier identified for this particular geometry will be inseparable.

7.3 An outlier of 4500m in SV17

If the outlier added in SV17 is changed to 4500m, it is found that the identified outlier indeed corresponds with the true outlier in SV17, as shown in Table 10.

Table 10: Outlier Statistics for an Outlier of 4500m in SV17

SV	4	8	9	11	15	17	26	28
w	14.544	133.689	-267.165	615.868	-170.904	706.314	-143.208	-706.264

If the separability of the outlier detection statistics are considered, using the separability statistics that are shown in Table 11. Then it is found that all the separability statistics, with respect to the identified outlier in SV17, pass at the significance level of 0.1%. Hence, the conclusion is made that SV17 can now be confidently rejected as the outlier.

If the separability using the reapplication of the global model test is considered, the statistics obtained are shown in Table 12. Hence, with a critical value of 7.815 at the significance level of 5%, it is observed that only the global model test of SV17 passes. Therefore, the conclusion is made that SV17 can be confidently rejected as the outlier.

Table 11: Separability Statistics for an Outlier of 4500m in SV17

J_{ik}		SV_k							
		4	8	9	11	15	17	26	28
SV_i	4		201.117	214.595	-431.379	-122.747	-494.494	-104.801	-491.283
	8	201.117		364.681	578.490	-28.972	-449.562	-7.919	-452.297
	9	-214.595	-364.681		513.942	-441.475	393.768	-357.374	393.836
	11	431.379	578.490	513.942		565.656	-178.675	562.653	-178.015
	15	-122.747	-28.972	-441.475	-565.656		434.703	-257.579	434.708
	17	494.494	449.562	393.768	178.675	434.703		446.216	3.865
	26	-104.801	-7.919	-357.374	-562.653	-257.579	446.216		445.005
	28	-491.283	-452.297	-393.836	-178.015	-434.708	3.865	-445.005	

Table 12: Global Model Statistics for an Outlier of 4500m in SV17

SV	$v^T Pv/{\sigma_0}^2$
4	498669.532
8	481012.225
9	427421.632
11	119361.969
15	469657.701
17	0.792
26	478363.736
28	71.575

7.4 The Minimally Separable Biases

To explain the reason for the outlier in SV17 requiring to be significantly larger than the MDB in order to be confidently separated from other measurements, the MSBs with the JN-Test can be examined. Therefore, calculating the MSBs via Equation (24), with the probabilities of committing Type I and II errors set to 0.1% and 20% respectively, the MSBs can be obtained as shown in Table 13. Hence, it can be seen that all the MSBs are larger than their MDBs shown in Table 1. However this is particularly the case with the MSB between SV17 and SV28 being in the order of thousands of meters. Hence, the reason for the bias in SV17 requiring to be 4500m, which is 5% bigger than its MSB of 4069m, before it could be confidently separated.

In addition, if the separability multiplying factors are computed from Equation (27) as shown in Table 14, it is also verified that all the MSBs are greater than their MDBs, since the values are bigger than one. In addition, the separability multiplying factor between SV17 and SV28 are also significantly larger. However, the reason for this can be explained by Equation (27) where it is seen that highly correlated outlier statistics result in large separability multiplying factors. Therefore, the MSBs and the separability multiplying factors are a reflection of the correlation coefficients in Table 3. This is particularly the case with the very high correlation between SV17 and SV28 resulting in the very large MSBs and separability multiplying factors.

The MSBs can also be evaluated from the multiple hypothesis method given by Förstner (1983) and Li

(1986). Therefore setting the accepted probability of committing a Type III error, as defined by Förstner (1983), to 0.1%, the separability multiplying factors can be obtained as shown in Table 15. Then by multiplying by the MDBs in Table 1, the MSBs can the obtained as shown in Table 16. From the MSBs and separability multiplying factors, it can also be seen that the MSBs are greater than their MDBs, which is particularly the case for SV17 and SV28. In addition, the multiple hypothesis method also confirms that a bias in SV17 is required to be greater than 3044m in order to be separated with the given confidence level.

Table 13: Minimally Separable Biases (MDBs) with the JN-Test

MSB_{ik} (m)		SV_k							
		4	8	9	11	15	17	26	28
SV_i	4		165.360	92.981	87.524	95.795	87.234	99.406	86.673
	8	150.795		101.244	85.875	86.627	87.369	92.577	87.908
	9	85.896	102.564		166.030	113.621	101.008	98.156	101.037
	11	62.972	67.752	129.308		63.117	173.292	65.074	172.753
	15	97.877	97.058	125.666	89.633		101.226	102.269	101.237
	17	37.713	41.420	47.269	104.128	42.831		41.803	4069.200
	26	106.292	108.551	113.612	96.712	107.028	103.392		103.120
	28	29.699	33.031	37.476	82.274	33.951	3225.185	33.045	

Table 14: Separability Multiplying Factors with the JN-Test

$k_{\rho_{ik}}$		SV_k							
		4	8	9	11	15	17	26	28
SV_i	4		2.710	1.524	1.435	1.570	1.430	1.629	1.421
	8			1.820	1.544	1.557	1.570	1.664	1.580
	9				2.946	2.016	1.792	1.742	1.793
	11					1.438	3.948	1.483	3.936
	15						1.624	1.641	1.624
	17							1.585	154.282
	26								1.581
	28								

Table 15: Multiple Hypothesis Method-based Separability Multiplying Factors

$k_{\rho_{ik}}$		SV_k							
		4	8	9	11	15	17	26	28
SV_i	4		2.027	1.000	1.000	1.001	1.000	1.093	1.000
	8			1.331	1.001	1.001	1.001	1.147	1.001
	9				2.202	1.501	1.302	1.248	1.305
	11					1.000	2.952	1.000	2.941
	15						1.082	1.112	1.082
	17							1.006	115.397
	26								1.001
	28								

Table 16: Multiple Hypothesis Method-based Minimally Separable Biases

MSB_{ik} (m)		SV_k							
		4	8	9	11	15	17	26	28
SV_i	4		123.639	61.033	61.012	61.043	61.010	66.659	61.008
	8	112.749		74.029	55.661	55.664	55.666	63.797	55.668
	9	56.379	74.988		124.086	84.571	73.390	70.327	73.543
	11	43.897	43.915	96.649		43.898	129.574	43.906	129.097
	15	62.358	62.355	93.526	62.327		67.442	69.305	67.442
	17	26.377	26.391	34.348	77.859	28.542		26.535	3043.570
	26	71.265	74.793	81.393	65.240	72.530	65.617		65.263
	28	20.905	20.917	27.280	61.482	22.622	2412.269	20.917	

When the MSBs from the two methods are compared, it can be seen that the MSBs from the multiple hypothesis method are less than the MSBs in Table 13. However, it appears that the reason for this is that the multiple hypothesis method is a combined test, and does not take into account for the probability of α_S. At this stage, it appears that the new separability measures are more conservative than the existing one and the computations of the probabilities used in the JN-test are much easier than those for the combined test. Further analysis on the differences between these two methods of separability test is needed.

8. Concluding Remarks

There has been an ever increasing demand to automate the detection, identification and separation of outlying measurements in data processing. However, in order to do so, practical methods to ensure that the identified outlying measurement can be confidently separated as the real outlier are required.

To address this issue, a separability test has been proposed based on the difference between two outlier detection statistics. If the difference between the outlier detection statistics is large enough to ensure that the probability of rejecting the wrong measurement is sufficiently small, it is concluded that the outlier statistics can be confidently separated. In addition, the MSB and separability multiplying factor associated with the new separability test (called **JN-Test**) have also been derived to enhance the analysis of the separability of a measurement system.

With the numerical examples, it has been shown that while an outlier is required to be larger than the MDB in order to be detected, it is not until the outlier is larger than the MSB that the outlier can be confidently separated, hence being automatically rejected. It has also been demonstrated that the difference between the MDB and the MSB is highly dependent on the correlation coefficients between the outlier detection statistics, with higher correlation coefficients causing the MSBs to be significantly larger than the MDBs.

When the proposed method was compared with the existing separability methods, it has been found that the proposed separability statistic appears to agree with the reapplication of the global model test method. However, the probability level at which the decision is made in the reapplication of the global model test method is unclear. In addition, the differencing outlier statistics method fails to account for the magnitude of the observed outlier statistics. In the comparison of the new separability test with the multiple hypothesis method it was discovered that the new MSB values are more conservative than the multiple hypothesis methods values. However, much of this may be due to the fact that the multiple hypothesis method is a combined single test, which does not consider the probability α_S used in the proposed separability test.

Further analysis is required to find out the reasons behind the minor discrepancies between the new MSBs and those obtained from the multiple hypothesis method. More broadly, though the outlier separability test can be considered as a liner model separability test, that also requires further extension to multiple dimensions, to deal with, for example, multiple outlier cases.

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Biography

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