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Receiver Autonomous Integrity Monitoring in Urban Vehicle Navigation: The Five Satellite Case

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Abstract

Receiver Autonomous Integrity Monitoring is most often described using an example where six pseudoranges estimate four unknowns. In this paper, the implications of using only five satellites are investigated. An earlier paper showing that least-squares estimations involving one degree of freedom with equally weighted observations always result in residuals with a value of ±1 is reviewed. The results from this previous work are generalized for the case of weighted observations and a priori knowledge of measurement variance. The new general result is that, when there is one degree of freedom, the standardized residuals always equal ± the square root of the estimate variance factor. This result is then demonstrated using an epoch of real data collected during a vehicle navigation test in an urban canyon where six and then five pseudorange observations are available.

Keywords: Residuals, Receiver Autonomous Integrity Monitoring

1. Introduction

Receiver Autonomous Integrity Monitoring (RAIM) in Global Navigation Satellite Systems (GNSS) is the process of identifying faults (also known as blunders, outliers, or biases) in GNSS observations based only on information available to the receiver. The most common implementation involves the statistical testing of the least-squares residuals of a single epoch solution and it has been shown that this approach is equivalent to what is known in the geodetic surveying field as the statistical reliability (of networks) problem (Baarda, 1968), or in statistics simply as the detection of outliers. In most applications (geodetic networks, fitting models to sample data, etc) the number of observations vastly exceeds the number of unknowns. Even in GNSS, with a good view of the sky, tracking 10 or more satellites is not uncommon and redundancy of observations is easily obtained. Many more studies have been conducted to show the benefits of even more observations obtained using additional GNSS, other RF ranging techniques, and other self-contained sensors. However, there are still many scenarios where sufficient GNSS observations to obtain a reliable solution are either not available, or only just barely available. One such example is vehicle navigation in urban canyons, where six satellites could be considered "good" availability.

The three main methods of receiver autonomous integrity monitoring and their equivalences are given in Brown (1992) and reviewed in both Brown (1993) and Kelly (1998). In summary they are:

1. Range comparison, in which arbitrarily selected redundant measurements are tested to see if they fit into the confidence region of an also arbitrarily selected unique solution

2. Least-squares residuals testing, where the sum of squares of the least-squares residuals is tested against a chi squared distribution with n – u degrees of freedom (where n is the number of observations, and u the number of unknowns, in this case u = 4, and finally

3. Parity methods, where the least-squares residuals are transformed from observation space into a residual or error space where the elements of the residuals vector are orthogonal to the columns of the design matrix.

In both methods 2 and 3 above, the test statistic is the magnitude of the residual vector and while it is obvious that regardless of the basis used, the residual vector should have the same magnitude (and thus be subjected to the same test statistic), what is not emphasized in any textbook treatment of this problem, other than Strang and Borre (1997), is that in this space the parity vector has a dimensionality equal to the degrees of freedom, (n – u).

In the standard tutorial case, n = 6, n – u = 2, and the parity space is two-dimensional, or in other words the residual vector, which is orthogonal to the state estimate, is two-dimensional since the state vector occupies a four-dimensional space within the six-dimensional observation space. The n = 6 example is easy to illustrate, and Brown (1993) states "… it is convenient to use the six-in-view case for tutorial purposes. The generalization to n = 5 or n > 6 is fairly obvious, so this is not discussed in detail", while Kelly (1998) observes that in the n = 4 case n – u = 0 and residuals cannot be obtained but then goes on to state "Therefore at least five satellite constellation is necessary for detection" which is true.

However in the n = 5 case, the parity space is now one-dimensional and the residual vector is a scalar whose magnitude is expected to follow the square root of a chi-squared distribution with one degree of freedom, in other words a normal distribution. Projecting the residual vector back into the observation space leads to the interesting result that if all the observations are equally weighted, then all of the residuals will be equal as well. More generally, if the observations are unequally weighted, though the residuals may have different values, the standardized residuals will have equal values and furthermore the standardized residuals will all be equal to the square root of the posteriori variance factor, which itself is equivalent to the magnitude of the residuals vector that is used as the test statistic. The first case (one degree of freedom with equally weighted independent observations) was discussed by Draper and Joiner (1984) but their work does not seem to have been noticed by the satellite navigation community.

The purpose of this paper is to demonstrate the problem of testing residuals for RAIM purposes with one degree of freedom when using single epoch least-squares for vehicle navigation.

The remainder of this paper is organized as follows. First the result of Draper and Joiner (1984) is presented using the notation of Leick (2004). Then this result is generalized to the case where the observation variance is known a priori, and then further generalized for unequally weighted observations. Finally a simple example of GPS pseudorange positioning will be shown with n = 5 and n = 6 for equally and unequally weighted cases with and without an artificially induced bias of 50 metres on one observation. These results are obtained using data collected in an urban canyon environment in Rome as part of the validation of the SCUTUM multipath mitigation algorithm (SCUTUM 2011). The implications of these results and their potential application of multi-GNSS scenarios with limited satellite availability are then discussed.

2. Least-squares residuals with one degree of freedom, equally weighted observations

The question of what happens to least-squares residuals with one degree of freedom was first addressed in a short paper in 1984 (Draper and Joiner, 1984). In this paper, a least-squares solution with equally weighted observations is presented and demonstrated with an example from an earlier paper (Fisher, 1938) where a seven parameter model is fitted to eight observations. Unfortunately this paper has rarely been cited. (No citations according to ISI Web of Science while Google Scholar finds two citations: One merely shows that the earlier paper by Fisher has been re-analyzed (Cox, 1984), the other is a textbook (Cotton, 1988) where a two-degree of freedom example is introduced with: "*Because Draper and Joiner (1984) gave evidence that ANOVA or t-tests with only one degree of freedom for error are not legitimate, the present design is…*".

The development below follows Draper and Joiner (1984) using the notation of Leick (2004) and is then generalized for the weighted observation case. The equations for linear least-squares are used, though the result can equally be applied to the linearized case.

The unweighted parametric least-squares problem involves the solution of the linear system of equations

$$\ell = \mathbf{A}\mathbf{x} + \mathbf{v} \qquad (1)$$

where ℓ is a vector of n observations with covariance matrix $\mathbf{Q}_\ell$ that is in this case identity, or at least a scalar a priori variance, σ_0^2, multiplied by identity, $\mathbf{Q}_\ell = \sigma_0^2\mathbf{I}$. $\mathbf{A}$ is the design matrix, $\mathbf{x}$ is a vector of u unknowns and $\mathbf{v}$ a vector of n residuals. The well-known solution is obtained by taking the pseudo-inverse of the design matrix as

$$\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{A})\mathbf{A}^T\ell \qquad (2)$$

From this the (estimated) residuals can be defined as the difference between observations and the adjusted observations $\hat{\ell} = \mathbf{A}\hat{\mathbf{x}}$

$$\mathbf{v} = \ell - \hat{\ell} = \ell - \mathbf{A}\hat{\mathbf{x}} \qquad (3)$$

Equation (3) can be re-written by substituting equation (2) for $\hat{\mathbf{x}}$:

$$\mathbf{v} = \ell - \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\ell = (\mathbf{I} - \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T)\ell \qquad (4)$$

where $\mathbf{I} - \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$ can be identified as the cofactor matrix of the estimated residuals $\mathbf{Q}_v$.

The equivalence of the parity method and the least-squares residual test method can be shown here, since the residuals can be shown to lie in a subspace with dimensions $n-u$ that is spanned by the columns of $\mathbf{Q_v}$. This subspace is orthogonal to the columns of $\mathbf{A}$ and to the columns of $\mathbf{A(A^TA)^{-1}A^T}$. This can be shown by pre-multiplying equation (3) by either $\mathbf{A(A^TA)^{-1}A^T}$ as in

$$\mathbf{A(A^TA)^{-1}A^T}(\ell - \mathbf{A}\hat{\mathbf{x}}) = 0 \tag{5}$$

or $\mathbf{A^T}$

$$\mathbf{A^T}(\ell - \mathbf{A}\hat{\mathbf{x}}) = 0 \tag{6}$$

In the parity method, analysis is conducted in the parity space where the components of the residual vector are orthogonal as opposed to the observation space where they are correlated. However in both methods, the magnitude of the vector is the same. When normalized by the degrees of freedom, the magnitude is also known as the a posteriori variance estimate $\hat{\sigma}_0^2$

$$\hat{\sigma}_0^2 = \frac{\mathbf{v^Tv}}{n-u} \tag{7}$$

If the original measurement variance was unknown, an estimate of the covariance matrix of the residuals $\mathbf{C_v}$ could be obtained as from the co-factor matrix as

$$\mathbf{C_v} = \hat{\sigma}_0^2\mathbf{Q_v} \tag{8}$$

Alternatively, though not shown by Draper and Joiner (1984), if the measurement variance *is* known, then the a priori value σ_0^2 , instead of the estimated variance, may be used in the residual covariance estimate

$$\mathbf{C_v} = \sigma_0^2\mathbf{Q_v} \tag{9}$$

Similarly the estimate of the a posteriori variance should be modified to include the known a priori measurement covariance matrix $\mathbf{Q}_l = \sigma_0^2\mathbf{I}$ such that

$$\hat{\sigma}_0^2 = \frac{\mathbf{v^TQ_\ell^{-1}v}}{n-u} = \frac{1}{\sigma_0^2}\frac{\mathbf{v^Tv}}{n-u} \tag{10}$$

in which case the estimate of the variance is now really an estimate of a variance factor. The variance factor should be equal to one in the case of correct choice of a priori measurement variance and bias-free residuals. In both cases the standardized residuals are defined as

$$\frac{v_i}{\sqrt{\{\mathbf{C_v}\}_{ii}}}, i = 1,2,...,n \tag{11}$$

Draper and Joiner (1984) then show that if $n-u=1$, and equations (7) and (8) are used to determine $\mathbf{C_v}$, then equation (11) evaluates to 1 or -1 for all n. Their proof is summarized in the following paragraph and equations (12) to (14).

Given that:

1. $\mathbf{v}$ lies in a space spanned by the columns of $\mathbf{Q_v}$ called the error space
2. The error space is orthogonal to both the columns of $\mathbf{A}$ (the parameter space) and the columns of $\mathbf{A(A^TA)^{-1}A^T}$ and
3. The error space is one-dimensional,

then the *i*th column of $\mathbf{Q_v}$, called $\mathbf{q}_i$ can be written as the residual vector times a scalar, namely $\mathbf{q}_i = a_i\mathbf{v}$. Then if the *i*th residual is computed from the transpose of this column $\mathbf{q}_i^T$ and the observation vector ℓ (following equation (4)) and $a_i\mathbf{v}$ is substituted for $\mathbf{q}_i$, and $\mathbf{v}+\hat{\ell}$ for ℓ,

$$\mathbf{v}_i = \mathbf{q}_i^\mathbf{T}\ell = a_i\mathbf{v^T}(\mathbf{v}+\hat{\ell}) = a_i\mathbf{v^Tv} = a_i\hat{\sigma}_0^2 \tag{12}$$

Then, since $\mathbf{Q_v}$ is symmetric and idempotent,

$$q_{ii} = \mathbf{q}_i^\mathbf{T}\mathbf{q}_i = a_i^2\mathbf{v^Tv} = a_i^2\hat{\sigma}_0^2 \tag{13}$$

Finally substituting equation (12) and (13) into equation (11) yields

$$\frac{v_i}{\sqrt{\hat{\sigma}_0^2 q_{ii}}} = \frac{a_i\hat{\sigma}_0^2}{\sqrt{\hat{\sigma}_0^2 a_i^2\hat{\sigma}_0^2}} = \pm 1, i = 1,2,...,n \tag{14}$$

If the a priori variance is known, $\mathbf{C_v}$ is instead obtained from equation (9) and $\hat{\sigma}_0^2$ defined as in equation (10) then equation (12) becomes

$$v_i = \mathbf{q}_i^\mathbf{T}\ell = a_i\mathbf{v^T}(\mathbf{v}+\hat{\ell}) = a_i\mathbf{v^Tv} = a_i\hat{\sigma}_0^2\sigma_0^2 \tag{15}$$

equation (13) evaluates to

$$q_{ii} = \mathbf{q}_i^\mathbf{T}\mathbf{q}_i = a_i^2\mathbf{v^Tv} = a_i^2\hat{\sigma}_0^2\sigma_0^2 \tag{16}$$

and substitution into equation (11) gives

$$\frac{v_i}{\sqrt{\sigma_0^2 q_{ii}}} = \frac{a_i\hat{\sigma}_0^2\sigma_0^2}{\sqrt{\sigma_0^2 a_i^2\hat{\sigma}_0^2\sigma_0^2}} = \pm\hat{\sigma}_0, i = 1,2,...,n \tag{17}$$

In other words the standardized residuals are all equal to ± the square root of the variance factor, which is also equal to the magnitude of the residual vector.

3. Weighted Case

In the weighted parametric case, the only difference is that the measurements are now weighted by the inverse of their covariance matrix $\mathbf{P} = \mathbf{C}_\ell^{-1}$ which is now not the identity matrix, nor the identity matrix multiplied by an a priori variance. The resulting solution is well known:

$$\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{P}\mathbf{A})\mathbf{A}^T\mathbf{P}\ell \qquad (18)$$

where

$$\mathbf{v} = \ell - \mathbf{A}(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}\ell = (\mathbf{P}^{-1} - \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T)\mathbf{P}\ell \qquad (19)$$

and

$$\mathbf{C}_\mathbf{v} = \mathbf{P}^{-1} - \mathbf{A}(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T \qquad (20)$$

Or, as also commonly expressed,

$$\mathbf{v} = \mathbf{C}_\mathbf{v}\mathbf{P}\ell \qquad (21)$$

In this case, the a posteriori variance factor $\hat{\sigma}_0^2$ is obtained from the weighted sum-of-squares of residuals

$$\hat{\sigma}_0^2 = \frac{\mathbf{v}^T\mathbf{P}\mathbf{v}}{n-u} \qquad (22)$$

and the standardized residuals continue to be defined as

$$\frac{v_i}{\sqrt{\{\mathbf{C}_\mathbf{v}\}_{ii}}}, i = 1, 2, ..., n \qquad (23)$$

The result is the same, though now equations (15), (16), and (17) must be re-evaluated retaining the weight matrix $\mathbf{P}$ explicitly, and using $\mathbf{C}_\mathbf{v}$ throughout as there is now no single a priori measurement variance to conveniently factor out.

$\mathbf{v}$ continues to lie in a space spanned by the columns of $\mathbf{C}_\mathbf{v}$ and is orthogonal to both the columns of $\mathbf{A}$ and the columns of $\mathbf{A}(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T$, however, the orthogonality (similar to distance in equation (22)) is now evaluated using $\mathbf{P}$ as a metric.

$$\mathbf{A}(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}(\ell - \mathbf{A}\hat{\mathbf{x}}) = 0 \qquad (24)$$

and

$$\mathbf{A}^T\mathbf{P}(\ell - \mathbf{A}\hat{\mathbf{x}}) = 0 \qquad (25)$$

The ith column of $\mathbf{C}_\mathbf{v}$, $\mathbf{c}_i$ can be written as, $\mathbf{c}_i = a_i\mathbf{v}$ and then

$$v_i = \mathbf{c}_i^T\mathbf{P}\ell = a_i\mathbf{v}^T\mathbf{P}(\mathbf{v} + \hat{\ell}) = a_i\mathbf{v}^T\mathbf{P}\mathbf{v} = a_i\hat{\sigma}_0^2 \qquad (26)$$

$$c_{ii} = \mathbf{c}_i^T\mathbf{c}_i = a_i^2\mathbf{v}^T\mathbf{P}\mathbf{v} = a_i^2\hat{\sigma}_0^2 \qquad (27)$$

$$\frac{v_i}{\sqrt{c_{ii}}} = \frac{a_i\hat{\sigma}_0^2}{\sqrt{a_i^2\hat{\sigma}_0^2}} = \pm\hat{\sigma}_0, i = 1, 2, ..., n \qquad (28)$$

If any of these three cases are applied to GPS navigation with five satellites, the only possible value for the standardized residual is either ±1 or $\pm\hat{\sigma}_0$ depending on the convention adopted for standardization (known vs. unknown a priori observation variance). This does not mean that five satellite geometries do not allow for fault detection, as the common value of the standardized residuals, which is the square root of the a posteriori variance factor and equal to the length of the residual vector and equal to the length of the parity vector as well, can be tested against a normal distribution, provided the a priori variance of the observations is known.

Fault isolation is another problem, and again, in most previous RAIM literature, it is approached from the assumption that fault isolation can only be accomplished in the case of six or more satellites since the usual method is to remove one observation and then test the magnitude of the residuals of the reduced solution. In Brown (1993) a parity method is described for fault isolation in the six satellite case, where each satellite is described as having a characteristic bias line in a two-dimensional parity space. In Brown and Sturza (1990) the equivalence between this method and testing the standardized residuals is shown. If the parity space is one-dimensional, then distinguishing biases becomes impossible. This is equivalent to saying that in the observation space, the standardized residuals are all equal, thus isolating the biased observation is not possible through analysis of the residuals alone.

4. Example

In order to demonstrate this effect, consider an example of a user located in an urban canyon tracking six satellites. The data and results presented here are obtained in an urban canyon in Rome as part of a test campaign designed to evaluate the performance of a system for tracking vehicles carrying dangerous goods (SCUTUM, 2011). The satellite elevations, azimuths, and pseudoranges are given in Table 1. This particular data epoch occurs at GPS week 1609, time of week 207211 and was collected using a Novatel OEMV2 mounted on a vehicle traveling in a deep urban environment. At this time, 12 satellites are above the horizon, but only six are being tracked due to signal shading.

Table 1: 6 GPS pseudorange observations

Satellite (PRN)	Elevation (°)	Azimuth (°)	Pseudorange (m)
12	16.14	115.41	24170384.00
21	44.88	182.40	22080190.44
25	52.88	109.57	21201116.93
29	70.99	27.59	20473347.87
30	83.36	30.16	19849599.86
31	52.07	276.24	21445379.88

The corresponding design matrix, with columns corresponding to latitude, longitude, height, and clock offset (in metres), for this epoch is:

$$A = \begin{bmatrix} -0.4122 & 0.8676 & 0.2781 & -1 \\ -0.7079 & -0.0296 & 0.7057 & -1 \\ -0.2021 & 0.5687 & 0.7973 & -1 \\ 0.2886 & 0.1509 & 0.9455 & -1 \\ 0.0999 & 0.0581 & 0.9933 & -1 \\ 0.0668 & -0.6110 & 0.7888 & -1 \end{bmatrix} \quad (29)$$

For simplicity, let us assume the point of expansion about the correct solution,

$$x_0 = [\phi, \lambda, h, cdt]^T = \begin{bmatrix} 41.90864916°\text{N} \\ 12.54108307°\text{E} \\ 84.159\text{ m} \\ -513.648\text{ m} \end{bmatrix} \quad (30)$$

The misclosure vector, $w = l - Ax_0$, is this case is

$$w = [0.48001, 0.99641, -2.38558, 1.54711, 0.51788, -1.15584]^T \quad (31)$$

Assuming equally weighted observations with unit variance, equation (18) evaluates to zero and the residuals are equal to the misclosure:

$$v = w - A\hat{x} = [0.48001, 0.99641, -2.38558, 1.54711, 0.51788, -1.15584]^T \quad (32)$$

The a posteriori variance factor is 5.4560 m^2 and the cofactor matrix of the residuals evaluates to

$$Q_v = \begin{bmatrix} 0.0474 & 0.0192 & -0.1391 & -0.0215 & 0.1483 & -0.0543 \\ & 0.1141 & -0.1858 & 0.2253 & -0.0706 & -0.1023 \\ & & 0.5659 & -0.2218 & -0.2765 & 0.2572 \\ & & & 0.5247 & -0.3547 & -0.1520 \\ & & & & 0.6250 & -0.0715 \\ & & & & & 0.1229 \end{bmatrix} \quad (33)$$

The standardized residuals (assuming the known unit measurement variance) are

$$v = [2.2058, 2.9494, -3.1711, 2.1359, 0.6551, -3.2971]^T \quad (34)$$

Now consider the case where the last observation is removed leaving only five pseudoranges. The design matrix is now

$$A = \begin{bmatrix} -0.4122 & 0.8676 & 0.2781 & -1 \\ -0.7079 & -0.0296 & 0.7057 & -1 \\ -0.2021 & 0.5687 & 0.7973 & -1 \\ 0.2886 & 0.1509 & 0.9455 & -1 \\ 0.0999 & 0.0581 & 0.9933 & -1 \end{bmatrix} \quad (35)$$

Assuming the same point of expansion, the misclosure vector is now

$$w = [0.48001, 0.99641, -2.38558, 1.54711, 0.51788]^T \quad (36)$$

Equation (18) evaluates to $\hat{x} = [4.8074, -7.8595, -12.1123, -12.6797]^T$ where each element has units of metres corresponding to a change easting, northing, vertical, and clock offset respectively. The residuals are

$$v = [-0.0309, 0.0345, 0.0336, 0.1175, -0.1547]^T \quad (37)$$

A second iteration substituting the residuals for the misclosure yields an additional change in $\hat{x}$ of less than 10^{-13} so the iteration is stopped. The a posteriori variance factor is now 0.0410 m^2 and the cofactor matrix of the residuals evaluates to

$$Q_v = \begin{bmatrix} 0.0233 & -0.0260 & -0.0254 & -0.0887 & 0.1167 \\ & 0.0290 & 0.0283 & 0.0988 & -0.1301 \\ & & 0.0276 & 0.0963 & -0.1268 \\ & & & 0.3367 & -0.4432 \\ & & & & 0.5834 \end{bmatrix} \quad (38)$$

The standardized residuals (assuming the known unit measurement variance) are

$$v = [-0.20248, 0.20248, 0.20248, 0.20248, -0.20248]^T \quad (39)$$

Each being exactly $\pm$ the square root of the variance factor $\sqrt{0.0410} = 0.20248$.

If the measurements are weighted, the results are similar. Suppose that the fourth and fifth measurement (which have larger residuals in the example above) are given a lower weight, for example a measurement variance of 2.0 m^2 instead of 1.0 m^2. Re-evaluating equation (18) gives slightly different solution, namely

$$\hat{x} = [4.7584,-7.8333,-12.0179,-12.5957]^T \quad (40)$$

and different residuals,

$$v = [-0.0161, 0.0180, 0.0175, 0.1224, -0.1611]^T \quad (41)$$

but in the end, the standardized residuals are all equal to $\pm$ the square root of the variance factor, in this case $\sqrt{0.0214} = 0.01461$.

Returning to the original six satellite unweighted example, now consider the case where the first pseudorange is biased by 50 m. In this case, the solution is now

$$\hat{x} = [20.0449, 8.7515, -81.4105, -70.9391]^T \quad (42)$$

and the residuals are

$$v = [2.8479, 1.9558, -9.3390, 0.4743, 7.9332, -3.8721]^T \quad (43)$$

Q_v remains as given by equation (33), and the standardized residuals are now

$$v = [13.0866, 5.7893, -12.4143, 0.6548, 10.0347, -11.0455]^T \quad (44)$$

In this case, the square root of the sum of squares of the residuals is 9.4156, so a global test on the magnitude of this vector would clearly fail and furthermore the largest standard residual corresponds to observation containing the blunder as expected when testing for outliers.

However, now consider what happens if this scenario is repeated, but this time with the sixth observation removed. Now the solution is

$$\hat{x} = [36.1500, -17.5784, -121.9875, -113.4168]^T \quad (45)$$

and the residuals are:

$$v = [1.1362, -1.2667, -1.2346, -4.3150, 5.6801]^T \quad (46)$$

Q_v is given by equation (38) , and the standardized residuals are now

$$v = [7.4366, -7.4366, -7.4366, -7.4366, 7.4366]^T \quad (47)$$

Each one being equal to $\pm$ the square root of the sum of square of the residuals $\sqrt{55.3033} = 7.4366$. Here, a global test on the magnitude of this vector would clearly fail, but isolating the bias would not be possible through evaluation of the standard residuals.

5. Application to Multi-GNSS Positioning

The above result is particularly important in the case of stand-alone multi-GNSS positioning in urban environments. When two or more GNSS are used without differential corrections, it is necessary to estimate an additional clock offset to account for the offset between the two system times. There have been numerous evaluations of the reliability advantages of using multiple GNSS (O'Keefe (2001), Verhagen (2002), Ochieng et al. (2002) among other). In most of these works, the conclusion is that the great number of new signals provided by using multiple GNSS will provide a large improvement in the ability to detect blunders through RAIM. However only Hewitson and Wang (2006) seem to have addressed this issue of multi-constellation GNSS in the case where a limited number of satellites are visible. They evaluate the ability to identify a blunder (what they call the separability) by evaluating the correlation coefficients of standardized residuals that are used as test statistics. They conduct a 24-hour simulation of GPS, Galileo and GLONASS RAIM and observe several epochs where only two satellites are available from one of the three systems and observe that the correlation-coefficient of two standardized residuals in these cases is unity, and as a result the blunder cannot be isolated.

The development in this paper presents a theoretical reason for these results. When there is a single degree of freedom all of the standardized residuals are identical (and fully correlated), making blunder detection in the new measurements impossible. If two observations from an additional GNSS are added to any solution from another GNSS they will both equally observe the additional clock offset, with one degree of freedom resulting in no ability to isolate blunders in the new system. Additionally, when two observations from the additional GNSS are added to a unique (four satellite) solution from the first GNSS, builder detection in the combined system will be possible, but isolating the error will not be.

6. Conclusion

The purpose of this paper was to demonstrate the difficulty of generalizing the standard description of receiver autonomous integrity monitoring using six satellites in view down to the five satellite case. In the five satellite case, there is a single redundant observation that while it allows a least-squares solution and the generation of least square residuals, leads to the interesting result that all standardized residuals are equal to the magnitude of the residuals vector making it impossible to apply tests on the standard residuals to determine which, if any, of the observations is an outlier. A proof of this, given originally by Draper and Joiner (1984) was reviewed herein and generalized to the weighted least-squares case with known a priori measurement variance. The result is the theoretical reason why residuals or parity-based fault exclusion cannot be applied to cases with only one degree of freedom, however, if the measurement variance is known, the magnitude of the residual vector can still be used to determine if a fault exists.

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Biography

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